

**TOOL:** The Bode plot rules for a pair of complex conjugate poles involve approximating

$$|H(j\omega)| = \frac{1}{\left| \left( \frac{j\omega}{\omega_0} \right)^2 + \frac{1}{Q} \left( \frac{j\omega}{\omega_0} \right) + 1 \right|}$$

which has a resonant peak for high  $Q$ .

In CTool "[FILTERS:BODE PLOTS:2-pole low-pass:PEAK RESPONSE FREQ DERIVATION](#)", the frequency at the peak is shown to be at a frequency slightly different than  $\omega_0$ .

$$\frac{\omega_{\max}}{\omega_0} = \sqrt{1 - \frac{1}{2Q^2}}.$$

Note that there is no resonant peak for  $Q < 1/\sqrt{2}$ .

In CTool "[FILTERS:BODE PLOTS:2-POLE LOW-PASS:PEAK RESPONSE MAGNITUDE DERIVATION](#)", the peak magnitude is shown to be  $Q$  scaled by a factor that is small for high  $Q$ .

$$|H(j\omega_{\max})| = \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}}.$$

For  $Q < 1$ , the peak of the resonance is 15% or less and, for  $Q < 0.8$ , the peak of the resonance is 2% or less. Thus, we may choose to ignore the peak. If not, the above mentioned CTools have tables of peak frequencies and magnitudes that may be used.

For  $Q = 1$ ,  $|H_{\max}|$  is 1.15 and  $\omega_{\max} / \omega_0 = 1/\sqrt{2} \doteq 0.707$ .

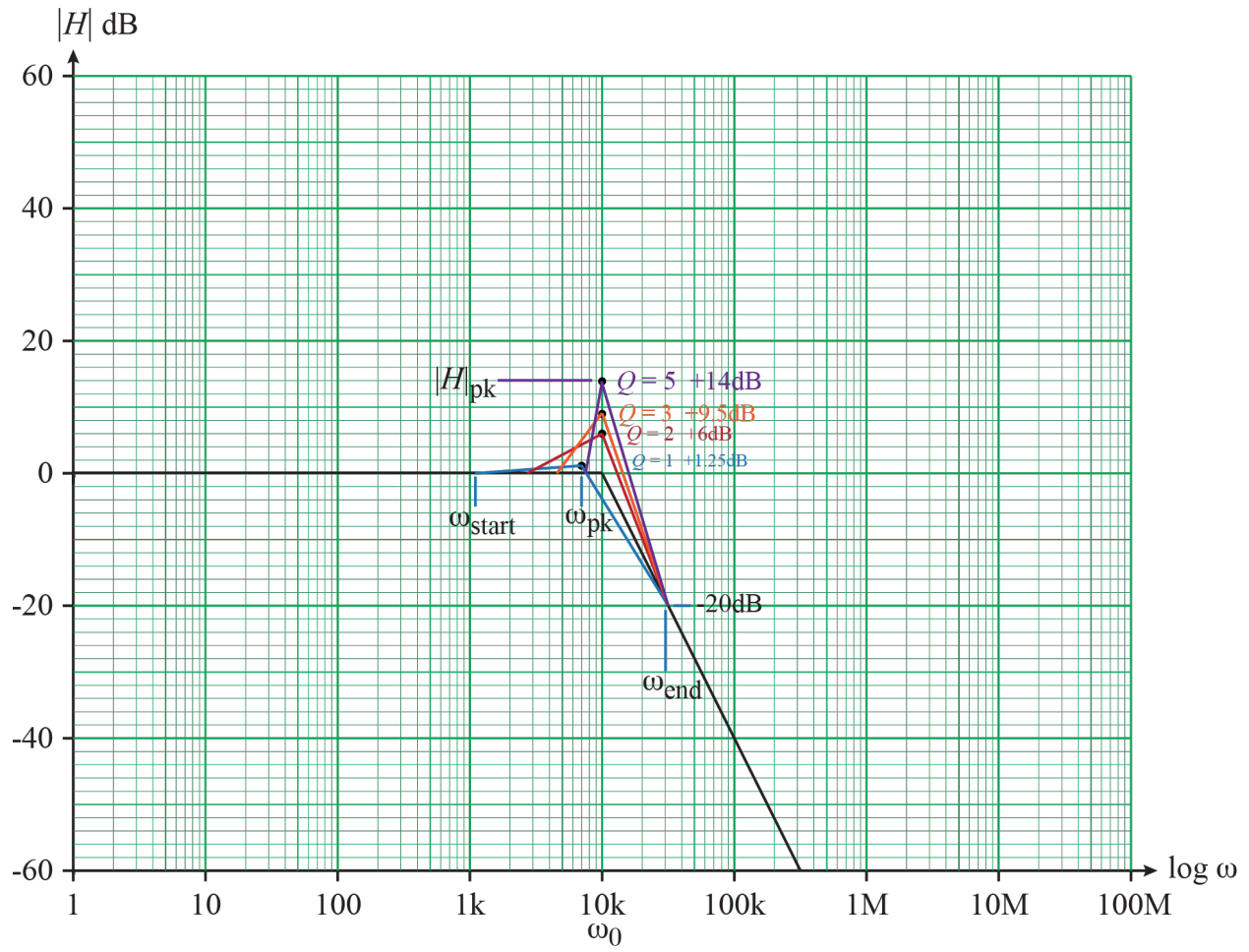
For  $Q \geq 2$ ,  $|H_{\max}| \approx Q$  is very accurate and  $\omega_{\max} / \omega_0 \approx 1$  with error  $< 7\%$  for  $Q = 2$ .

An approximation for the starting frequency for a line on the Bode plot that ends at the resonant peak gives a frequency in the decade before  $\omega_0$ .

$$\frac{\omega_{\text{start}}}{\omega_0} \approx \begin{cases} 0.15Q & 1 \leq Q \leq 6.5 \\ 1 & Q > 6.5 \end{cases}$$

A line from the resonant peak (or the from 0dB at  $\omega_0$  for low  $Q$ ) to a point at -20dB and  $\sqrt{10}\omega_0$  is a good approximation for all  $Q$  values.

$$\omega_{\text{end}} = 10\omega_0 \text{ and } -20\text{dB}$$



$$Q \geq 2, |H_{\text{max}}| \approx Q \text{ and } \omega_{\text{max}} / \omega_0 \approx 1$$

$$\frac{\omega_{\text{start}}}{\omega_0} \approx \begin{cases} 0.15Q & 1 \leq Q \leq 6.5 \\ 1 & Q > 6.5 \end{cases}$$

$$\omega_{\text{end}} = 10\omega_0 \text{ and } -20 \text{ dB}$$