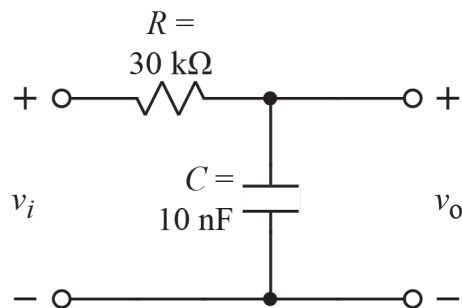
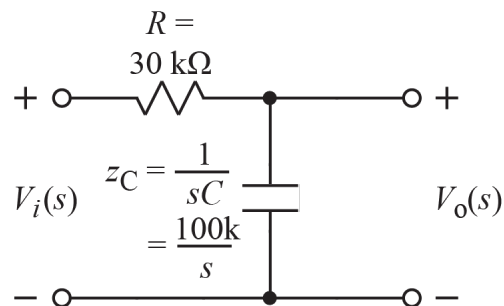


**EX:** An engineer plans to use a low-pass filter to smooth out a stair-stepped output from an Analog-to-Digital converter. The cutoff frequency of the filter is about 3.4 kHz to match old telephones. Note, however, that old phones have higher-order filters (a fourth-order Butterworth, for example [1]), so our low-pass filter rolls off more slowly than the actual filters found in old phones.



Find the magnitude response plot and phase response plot for the above filter using distances and angles from poles and zeros to a point at  $j\omega$  on the imaginary axis.

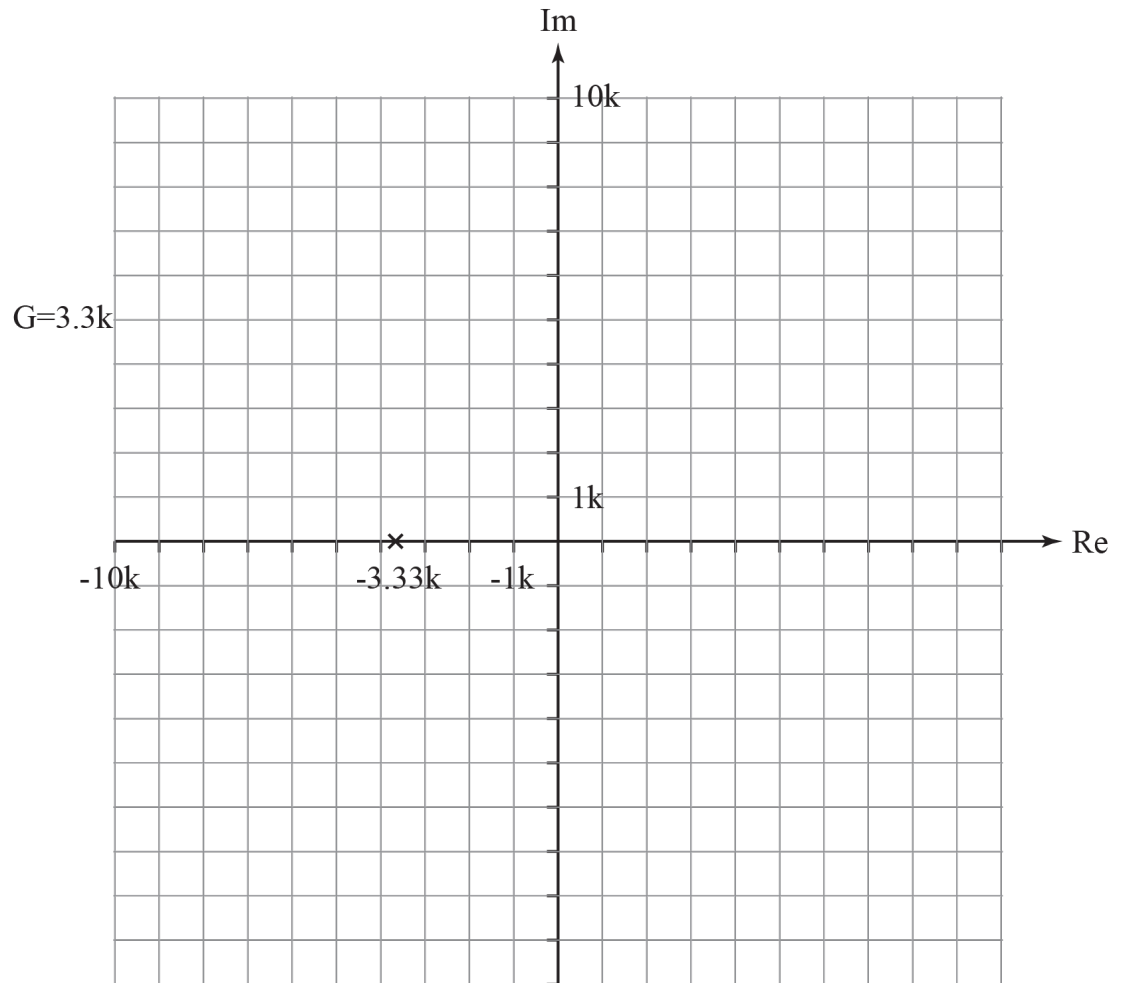
**SOL'N:** First, we create an  $s$ -domain model of the circuit.



The filter transfer function comes from the  $V$ -divider formula.

$$H(j\omega) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1 + sRC} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}} = \frac{3.3k}{s + 3.3k}$$

We have a single pole at  $-3.3 \text{ kr/s}$ , as shown on the following pole-zero plot.



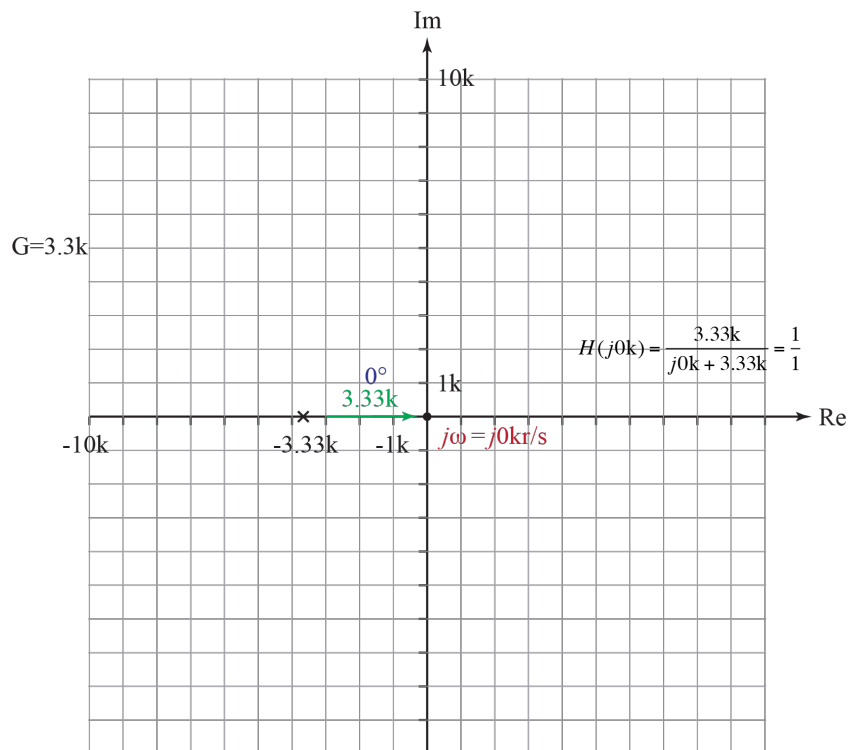
Now we use  $s = j\omega$ , meaning we move up the imaginary axis, and we find the magnitude and phase of vectors from the pole at  $s = -3.33k/s$  to  $j\omega$ . Graphically,  $j\omega - -\frac{1}{RC}$ , is the vector from the pole to  $j\omega$ . In equation form, we have the following expressions for magnitude and phase.

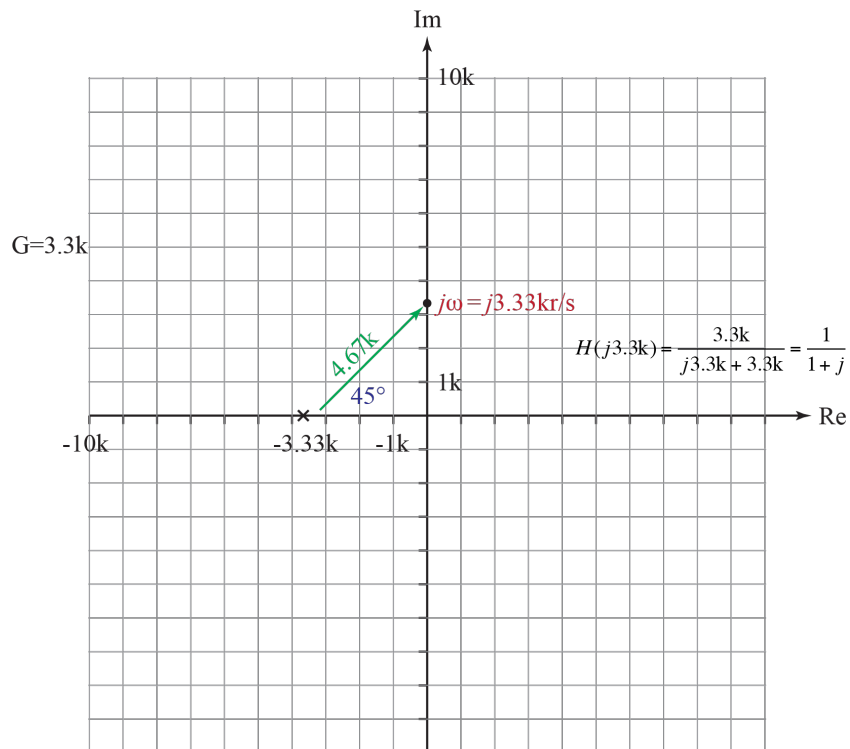
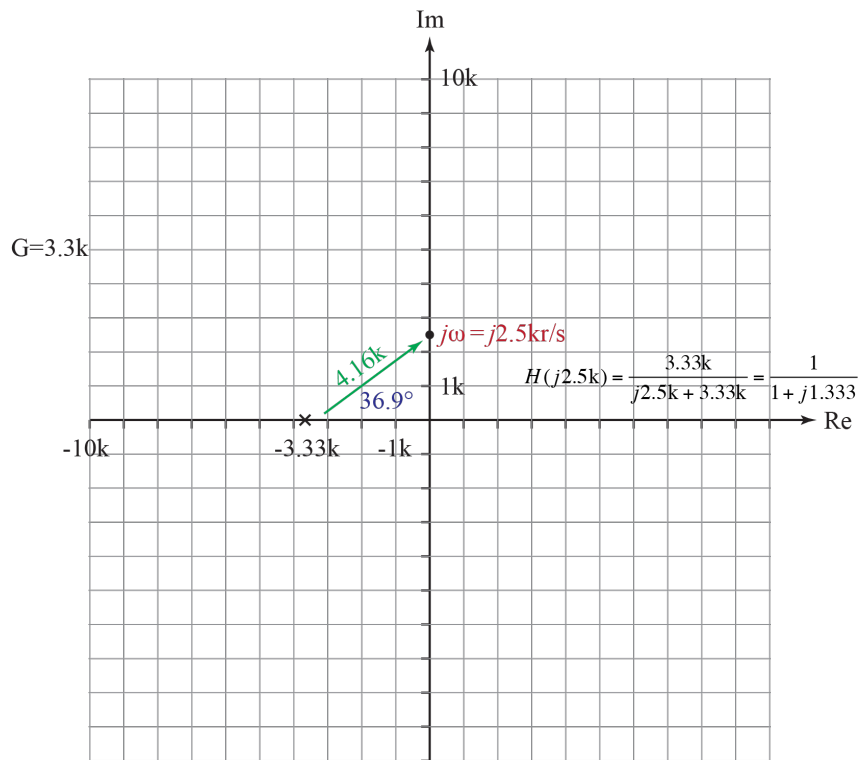
$$|H(j\omega)| = \frac{\left| \frac{1}{RC} \right|}{\left| j\omega - -\frac{1}{RC} \right|} = \frac{3.3k}{|j\omega - -3.3k|}$$

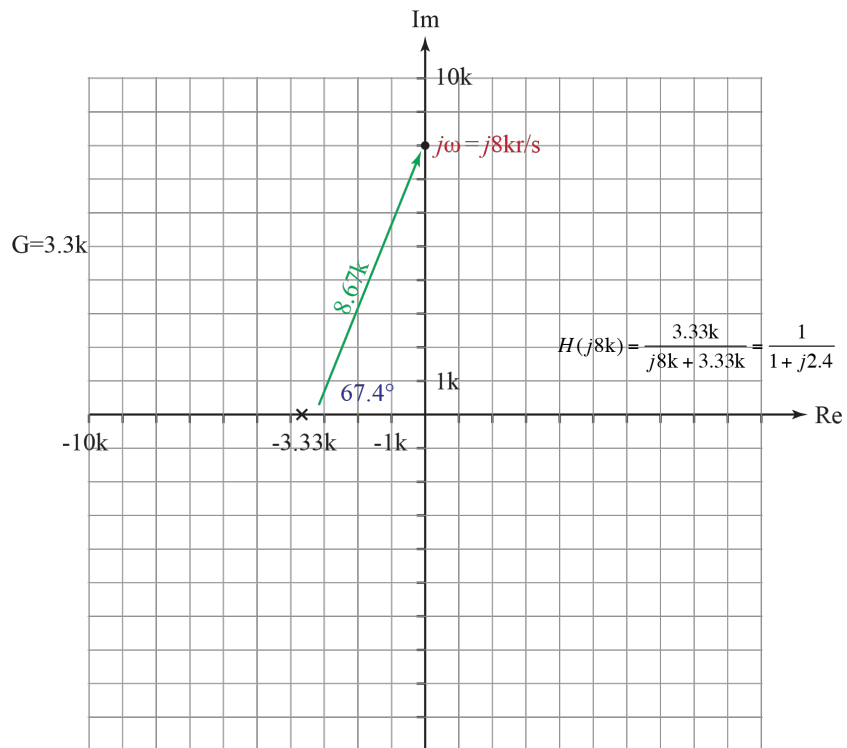
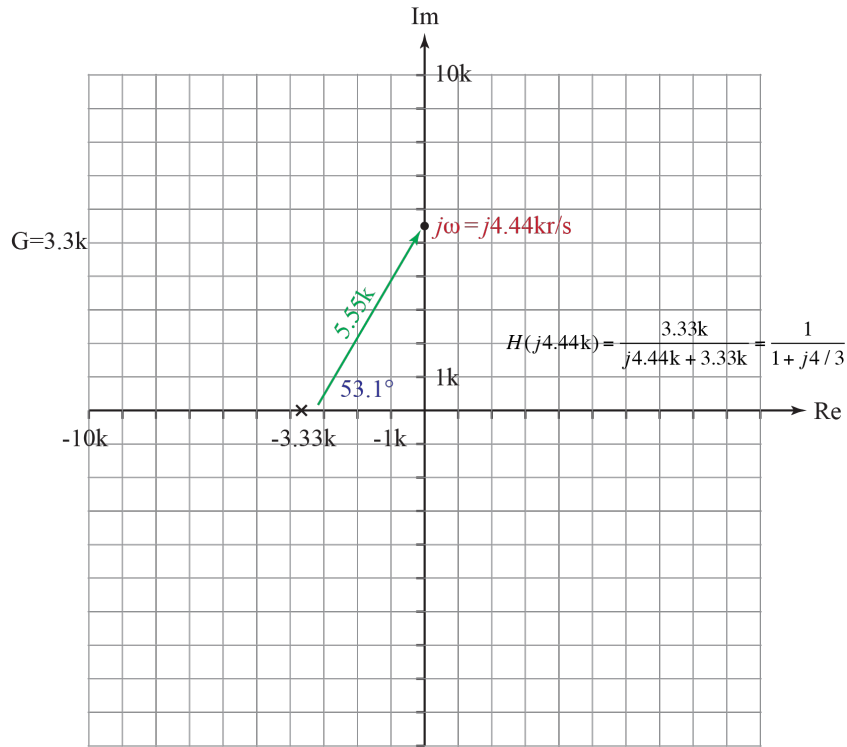
and

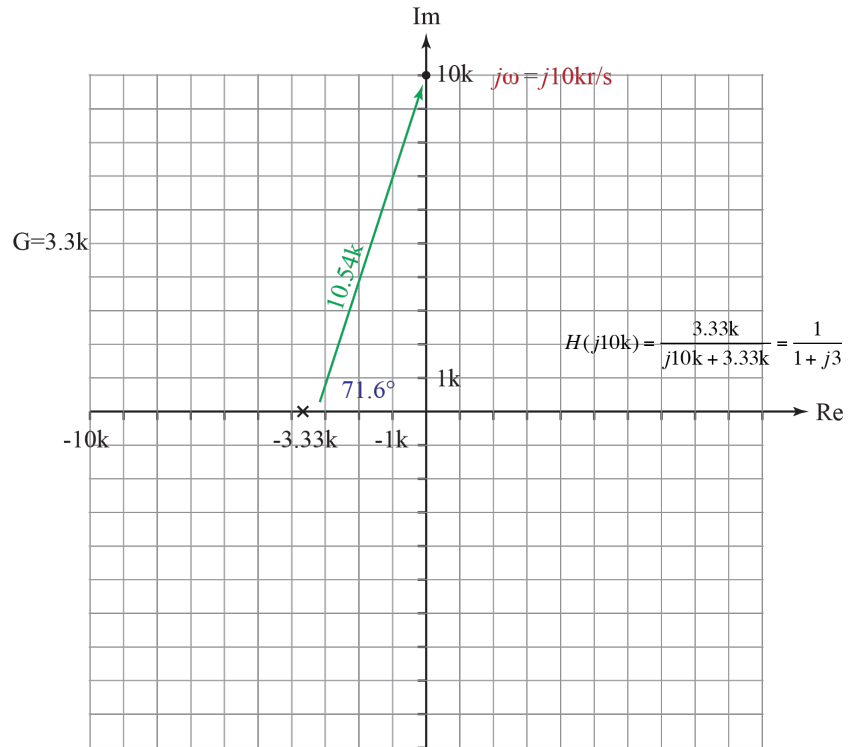
$$\angle H(j\omega) = \frac{\angle \frac{1}{RC}}{\angle \left( j\omega - \frac{1}{RC} \right)} = 0^\circ - \angle \left( j\omega - \frac{1}{RC} \right)$$

The following diagrams show the magnitude and phase angle of some of the vectors.

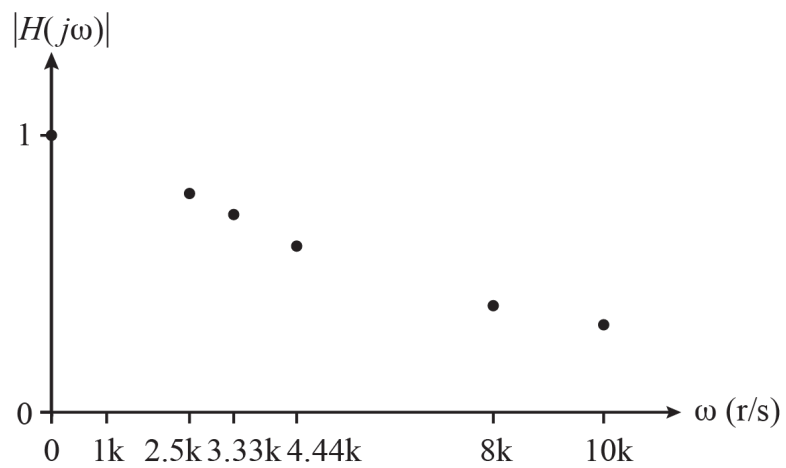




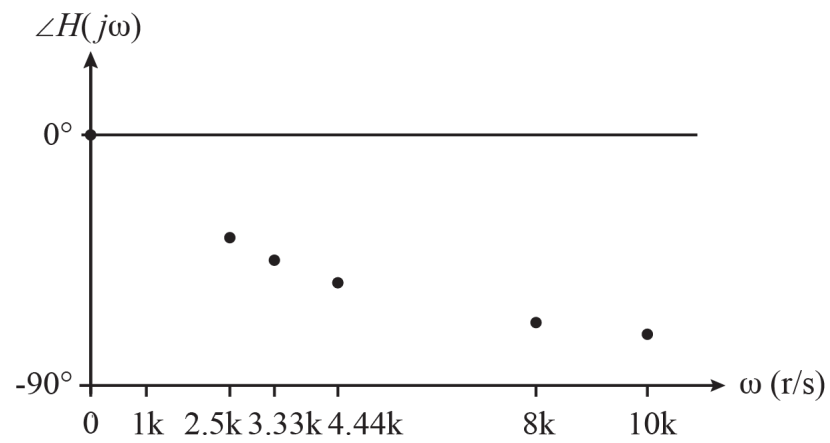




For the magnitude plot, we divide 3.33k by the length of the vector in the above figures.



For the phase plot, we take the negative of the angle of the vector in the above figures.



In practice, the purpose of the above process is to make a quick sketch of the magnitude or phase plot rather than a detailed and perfectly accurate plot. The process also ties the positions of the poles and zeros to magnitude and phase responses.

- REF:** [1] S. Oksanen and V. Välimäki, "Digital modeling of the vintage telephone sound," Proc of Internat. Computer Music Conf. 2011, U of Huddersfield, UK,  
[https://www.researchgate.net/publication/263007860\\_Digital\\_Modeling\\_of\\_the\\_Vintage\\_Telephone\\_Sound](https://www.researchgate.net/publication/263007860_Digital_Modeling_of_the_Vintage_Telephone_Sound), 31 July - 5 Aug 2011.