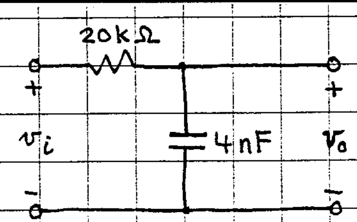


a) Find cutoff frequency (Hz) of above low-pass filter.

b) Calculate $H(j\omega)$ at $\omega_c$, $0.2\omega_c$, $8\omega_c$.

c) Given $v_i = 480 \cos \omega t$ mV find steady-state $v_o$ for $\omega = \omega_c, 0.2\omega_c, 8\omega_c$.

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ans: a) $f_c = 1990$ Hz

b) $H(j\omega_c) = \frac{1}{\sqrt{2}} \angle -45°$, $H(j\,0.2\omega_c) = 0.98 \angle -11.3°$, $H(j8\omega_c) = 0.124 \angle -82.9°$

c) $v_o(\omega_c) = 340 \cos(12.5\text{k}t - 45°)$ mV

$v_o(0.2\omega_c) = 471 \cos(2.5\text{k}t - 11.3°)$ mV

$v_o(8\omega_c) = 59.5 \cos(100\text{k}t - 82.9°)$ mV

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sol'n a) Find the transfer function $H(s = j\omega) \equiv \dfrac{V_o(s=j\omega)}{V_i(s=j\omega)}$.

We have a voltage divider: (using phasor $V$'s and impedances, $z$'s)

$$V_o = V_i \frac{z_C}{z_C + z_R} \quad \text{where} \quad z_C = \frac{1}{j\omega C} \quad z_R = R$$

$$\therefore\ H(j\omega) = \frac{V_o}{V_i} = \frac{z_C}{z_C + z_R} = \frac{1/j\omega C}{1/j\omega C + R} = \frac{1}{1 + j\omega RC}$$

The cutoff freq $\omega_c$ is the value of $\omega$ such that:*

1) $|H(j\omega_c)| = \frac{1}{\sqrt{2}} \cdot \max\limits_{\omega} |H(j\omega)|$; i.e. $\omega_c = \omega$ where magnitude gain is $\frac{1}{\sqrt{2}}$ times max magnitude gain.

2) The imaginary part of $H(j\omega)$ = real part of $H(j\omega)$;
i.e. $\text{Re}[H(j\omega_c)] = \text{Im}[H(j\omega_c)]$

3) The phase angle of $H(j\omega)$ is $\pm 45°$;
i.e. $\angle H(j\omega_c) = \pm 45°$

*applies to RL or RC low-pass or high-pass filter

We use the 2nd condition $\mathrm{Re}[H(j\omega_c)] = \mathrm{Im}[H(j\omega_c)]$.

It turns out that $\mathrm{Re}[H(j\omega_c)] = \mathrm{Im}[H(j\omega_c)]$ when the real part of the denominator equals the *imaginary* part of the denominator.

$$\mathrm{Re}[1+j\omega RC] = 1 \qquad \mathrm{Im}[1+j\omega RC] = \omega RC$$

$$\therefore \ \omega_c RC = 1 \quad \text{or} \quad \omega_c = 1/RC = 1/20k\cdot 4n = 1/80\mu$$

$$\text{or } \omega_c = 1M/80 = 1k \cdot 1k/80 = 12.5k \text{ rad/s}$$

$$\omega_c = 2\pi f_c \quad \Rightarrow \quad f_c = \frac{\omega_c}{2\pi} = \frac{12.5k}{2\pi} \text{ Hz} = 1990 \text{ Hz}$$

b) Use $H(j\omega) = \dfrac{1}{1+j\omega RC}$ from (a).

$$H(j\omega_c) = \frac{1}{1+j\omega_c RC} = \frac{1}{1+j\frac{1}{RC}RC} = \frac{1}{1+j}$$

$$\text{"} = \frac{1}{1+j}\,\frac{1-j}{1-j} = \frac{1-j}{1^2+1^2} = \frac{1}{2} - j\frac{1}{2}$$

$$\text{"} = \sqrt{\frac{1}{2}^2 + \left(\frac{-1}{2}\right)^2} \angle \tan^{-1}\frac{-1/2}{1/2} = \frac{1}{\sqrt{2}} \angle -45^\circ$$

or $H(j\omega_c) = \dfrac{1}{1+j} = \dfrac{1}{\sqrt{2}\angle 45^\circ} = \dfrac{1}{\sqrt{2}} \angle -45^\circ$
(alternate approach)

$$H(j0.2\omega_c) = \frac{1}{1+j\frac{0.2}{RC}\cdot RC} = \frac{1}{1+j0.2} = \frac{1}{\sqrt{1^2+0.2^2}\angle \tan^{-1}\frac{0.2}{1}}$$

$$\text{"} = \frac{1}{1.02\angle 11.3^\circ} = 0.98\angle -11.3^\circ$$

$$H(j8\omega_c) = \frac{1}{1+j\frac{8}{RC}RC} = \frac{1}{1+j8} = \frac{1}{\sqrt{1^2+8^2}\angle \tan^{-1}\frac{8}{1}}$$

$$\text{"} = \frac{1}{8.06\angle 82.9^\circ} = 0.124\angle -82.9^\circ$$

c) In terms of phasors, $V_o = V_i \cdot H(j\omega)$.
Thus, we convert $v_i(t)$ to phasor $V_i(j\omega)$:

$$v_i(t) = 480 \cos \omega t \text{ mV} \Rightarrow V_i(j\omega) = 480\angle 0^\circ \text{ mV}$$

$$V_o(j\omega_c) = 480\angle 0^\circ \text{ mV} \cdot \underbrace{\frac{1}{\sqrt{2}}\angle -45^\circ}_{H(j\omega_c)} = 339 \angle -45^\circ \text{ mV}$$

$$\therefore v_o(t) = 339 \cos(\omega t - 45^\circ) \text{ mV}$$

Note: Phasor multiplication rule is $R_1\angle\theta_1 \cdot R_2\angle\theta_2 = R_1R_2\angle\theta_1+\theta_2$.

$$V_o(j0.2\omega_c) = 480\angle 0^\circ \text{ mV} \cdot 0.98\angle -11.3^\circ = 470 \angle -11.3^\circ \text{ mV}$$

$$\therefore v_o(t) = 470 \cos(\omega t - 11.3^\circ) \text{ mV}$$

Note: Phasor to time domain conversion is $R\angle\theta \Rightarrow R\cos(\omega t + \theta)$.

$$V_o(j8\omega_c) = 480\angle 0^\circ \text{ mV} \cdot 0.124\angle -82.9^\circ = 59.5\angle -82.9^\circ \text{ mV}$$

$$\therefore v_o(t) = 59.5 \cos(\omega t - 82.9^\circ) \text{ mV}$$