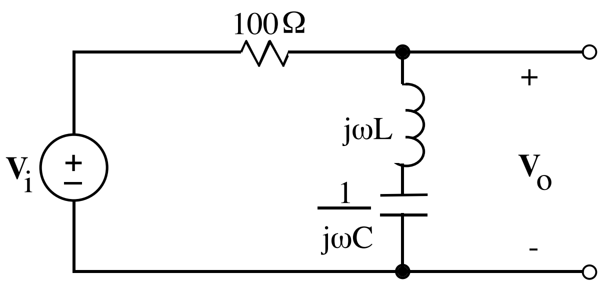
EX:



a. Choose values of L and C that will produce an ω_o of $2\pi \cdot 10^4$ and a Q of 2.

b. Calculate β, ω_{c1}, and ω_{c2}.

ANS: a) $L = 3.2$ mH, $C = 80$ nF

b) $\beta = 31.4$ k rad/s, $\omega_{C1} = 49.1$ k rad/s, $\omega_{C2} = 80.5$ k rad/s

SOL'N: a) This is a band reject filter with the following output:

$$\mathbf{V}_O = \mathbf{V}_i \underbrace{\frac{j\omega L + \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}}_{H(j\omega)} \quad \text{(V-divider)}$$

The series L and C will act like a wire at the resonant frequency ω_o and an open circuit for $\omega = 0$ (where C acts like an open circuit) and $\omega \to \infty$ (where L acts like an open circuit):

$$|H(j\omega)| = \begin{cases} \infty / \infty = 1 \text{ at } \omega = 0 \text{ or } \omega \to \infty \\ 0 \text{ at } \omega = \omega_o \text{ where } j\omega L = \dfrac{-1}{j\omega C} \end{cases}$$

The resonant frequency is found, as always, by solving for the frequency, ω_o, where the impedance of the L plus the impedance of the C equals zero:

$$\omega_o = \sqrt{\frac{1}{LC}}$$

From the course text, we have an equation for the Q of this particular filter circuit:

$$Q = \sqrt{\frac{L}{R^2C}}$$

(If necessary, we could also find Q by determining the cutoff frequencies where $|H(j\omega)| = 1/\sqrt{2}$. The difference of the cutoff frequencies is the bandwidth, β, and $\omega_o/\beta = Q$.)

Using the equations for ω_o and Q, we do some algebra to find C:

$$\omega_o Q = \sqrt{\frac{1}{LC}}\sqrt{\frac{L}{R^2C}} = \frac{1}{RC} \qquad \text{or} \qquad C = \frac{1}{R\omega_o Q}$$

Plugging in values given in the problem, we have

$$C = \frac{1}{100\Omega \cdot 2\pi \cdot 10^4/\text{s} \cdot 2} = \frac{1\text{F}}{4\pi \cdot 10^6} = \frac{1000}{4\pi}\text{nF} = 79.6 \text{ nF} \approx 80 \text{ nF}$$

Rearranging the equation for Q and using this value of C gives the value for L:

$$Q = \sqrt{\frac{L}{R^2C}} \Rightarrow L = Q^2R^2C = 4 \cdot 100^2 \cdot 80\text{n} = 320 \cdot 10 \text{ knH}$$

or

$$L = 3200 \ \mu\text{H} = 3.2\text{mH}$$

SOL'N: b) From the course text or calculations of cutoff frequencies as described above, we have equations for cutoff frequencies that apply to simple RLC bandpass and bandreject filters:

$$\omega_{C1} = \omega_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

$$\omega_{C2} = \omega_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

Summing the equations gives a formula for bandwidth, β:

$$\beta \equiv \omega_{C2} - \omega_{C1} = \frac{\omega_o}{Q}$$

$$\therefore \quad \beta = \frac{2\pi \cdot 10^4}{2} = \pi \cdot 10^4 \text{ rad/s} = 31.4 \text{ k rad/s}$$

Now we compute ω_{C1}, ω_{C2}:

$$\frac{1}{2Q} = \frac{1}{4}, \qquad \sqrt{1 + \left(\frac{1}{2Q}\right)^2} = \sqrt{\frac{17}{16}} = \frac{\sqrt{17}}{4}$$

$$\omega_{C1} = 2\pi \cdot 10^4\left(-\frac{1}{4} + \frac{\sqrt{17}}{4}\right) = \left(\frac{\sqrt{17} - 1}{2}\right)\pi \cdot 10^4 = 49.1 \text{ k rad/s}$$

$$\omega_{C2} = 2\pi \cdot 10^4\left(+\frac{1}{4} + \frac{\sqrt{17}}{4}\right) = \left(\frac{\sqrt{17} + 1}{2}\right)\pi \cdot 10^4 = 80.5 \text{ k rad/s}$$

Consistency check:

$$\sqrt{\omega_{C1} \cdot \omega_{C2}} = \sqrt{49.1 \text{ k} \cdot 80.5 \text{ k}} = 62.9 \text{ k} = 2\pi \cdot 10^4 = \omega_o \quad \checkmark$$