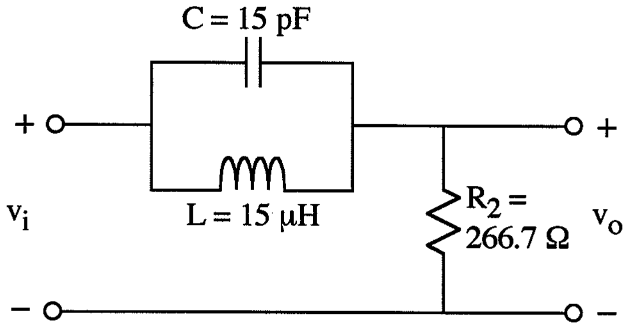
Ex:



For the band-reject filter shown above, calculate the following quantities:

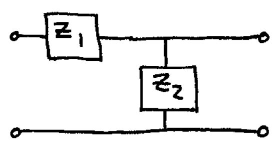
a) ω_0

b) f_0

c) ω_{C1} and ω_{C2}.

d) β and Q

sol'n: a) $\omega_0^2 = \dfrac{1}{LC}$ for RLC filters of simple form:



Z_1	Z_2
R	$\frac{1}{sC} + sL$
R	$\frac{1}{sC} \parallel sL$
$\frac{1}{sC} + sL$	R
$\frac{1}{sC} \parallel sL$	R

$$\omega_0^2 = \frac{1}{15\mu H \cdot 15 pF} = \frac{1}{(15 ns)^2}$$

$$\omega_0 = \frac{1}{15ns} = 66.7 \text{ M r/s}$$

b) $f = \dfrac{\omega}{2\pi}$ $\qquad f_0 = \dfrac{\omega_0}{2\pi} = \dfrac{66.7 \text{ Mr/s}}{2\pi} = 10.6 \text{ M r/s}$

c) $\omega_{C1,C2}$ are the frequencies where the magnitude of $H(s)$ is reduced by a factor of $\sqrt{2}$ relative to its max.

$$H(s) = \frac{R_2}{R_2 + sL \| \frac{1}{sC}}$$

$$sL \| \frac{1}{sC} = \frac{1}{\frac{1}{sL} + \frac{1}{\frac{1}{sC}}} = \frac{1}{\frac{1}{sL} + sC} = \frac{1}{j\left(\omega C - \frac{1}{\omega L}\right)}$$

$$H(s) = \frac{R_2}{R_2 - \frac{j}{\omega C - \frac{1}{\omega L}}}$$

We write $H(s)$ in form $H(s) = k \cdot \frac{1}{1+jX}$

where $k \equiv$ real and $X \equiv$ real.

$$H(s) = \frac{\cancel{R_2}}{\cancel{R_2}} \frac{1}{1 - j \frac{1}{R_2} \cdot \frac{1}{\omega C - \frac{1}{\omega L}}}$$

$$\max_{\omega} |H(s)| = 1 \quad \text{when } \omega = 0 \quad (L = \text{wire})$$
$$\text{or } \omega \to \infty \quad (C = \text{wire})$$

Thus, $|H(j\omega_c)| = \frac{1}{\sqrt{2}} \cdot 1$

It follows that $\frac{1}{\left| 1 - j \frac{1}{R_2} \cdot \frac{1}{\omega_c C - \frac{1}{\omega_c L}} \right|} = \frac{1}{\sqrt{2}}$

or $1 - j \frac{1}{R_2} \frac{1}{\omega_c C - \frac{1}{\omega_c L}} = 1 \pm j$

or $R_2 \left(\omega_c C - \frac{1}{\omega_c L} \right) = \pm 1$

We rewrite this as a quadratic eq'n.

Multiply both sides by $\frac{\omega_c}{RC}$, move $\pm\frac{\omega_c}{RC}$ to left.

$$\omega_c^2 \pm \frac{\omega_c}{RC} - \frac{1}{LC} = 0$$

$$\omega_{c1,2} = \pm\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

We must use $+\sqrt{\ }$ since ω_c values must be > 0.

$$\frac{1}{2RC} = \frac{1}{2\cdot 266.7\Omega\cdot 15\text{pF}} = \frac{1}{2\cdot\frac{8}{\cancel{3}}\cdot 100\Omega\cdot 5\cdot\cancel{3}\text{ pF}}$$

$$\text{"} = \frac{1\text{G}}{8} = 125\text{ Mr/s} = 25\cdot\frac{15}{3}\text{ Mr/s}$$

$$\frac{1}{LC} = \omega_0^2 = (66.7\text{ Mr/s})^2 = \left(\frac{200}{3}\text{ Mr/s}\right)^2$$

$$\sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} = \sqrt{125^2 + 66.7^2}\text{ Mr/s}$$

$$\text{"} = \sqrt{\left(25\cdot\frac{15}{3}\right)^2 + \left(25\cdot\frac{8}{3}\right)^2}\text{ Mr/s}$$

$$\text{"} = \frac{25\cdot 17}{3}\text{ Mr/s}$$

$$\omega_{c1} = \frac{25}{3}(-15+17)\text{ Mr/s} = \frac{50}{3}\text{ Mr/s}$$

$$\omega_{c2} = \frac{25}{3}(15+17)\text{ Mr/s} = \frac{800}{3}\text{ Mr/s}$$

$$\omega_{c1} = 16.7\text{ Mr/s} \qquad \omega_{c2} = 266.7\text{ Mr/s}$$

d) $$\beta = \omega_{c2} - \omega_{c1} = \frac{1}{RC} = 2\cdot\frac{1}{2RC} = 250\text{ Mr/s}$$

$$Q = \frac{\omega_0}{\beta} = \frac{66.7\text{ Mr/s}}{250\text{ Mr/s}} = 0.267$$