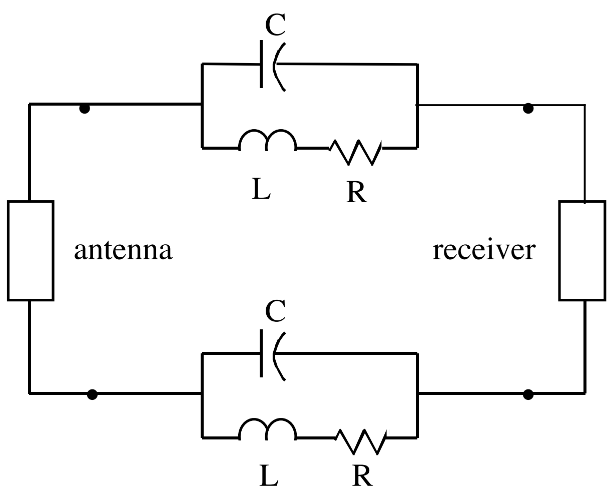
EX: The circuit shown below is a wave trap, used to prevent the signal from an amateur radio transmitter from entering the input of a TV receiver. Choose proper values of L and C if the transmitter frequency is 50 MHz, and R does not affect the resonant frequency appreciably. Explain how the circuit works. The bottom edge of Channel 2 is at 54 MHz.

What value of R would be required to make |Z| at 54 MHz of one of the resonant circuits equal to 1/10 of its value at resonance and what value of impedance would that be at 54 MHz?

Note: the frequencies here are in units of Hz rather than rad/s.



ANS: There is no unique answer. For C = 100 pF, then L = 0.101 μH, R ≈ 0.492 Ω, |Z| ≈ 205 Ω at 54 MHz.

SOL'N: The idea is that L parallel C will act like an open circuit at resonance. This prevents the interfering signal from reaching the receiver. With the R included, (but relatively small), the resonant frequency remains approximately the same:

$$\omega_o \equiv \frac{1}{\sqrt{LC}}$$

In units of Hz, we have

$$f_o \equiv \frac{\omega_o}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

Any L and C satisfying the following equation will yield the correct resonant frequency:

$$LC = \frac{1}{(2\pi f_o)^2} = \frac{1}{\left(2\pi 50 \cdot 10^6\right)^2} \text{ s}^2 \approx 10.1 \cdot \mu\text{ps}^2$$

One solution is C = 100 pF and L = 0.101 μH.

Now we find the value for R.

At resonance, we have equal but opposite reactances for the L and C.

Define the reactances at ω_o to be +jX and -jX. From the definition of ω_o, we have X = √(L/C) = 31.8 Ω.

At ω_o, the impedance, z_o, of one side of the trap circuit, (i.e., one C in parallel with an L plus an R), is

$$z_o = \frac{(R + jX)(-jX)}{R + jX - jX} = \frac{X^2 - jRX}{R} = X\left(\frac{X}{R} - j\right).$$

For a frequency, $k\omega_o$, our L and C reactances become jkX and –jX/k, and we have

$$z_k = \frac{(R + jkX)(-jX/k)}{R + jkX - jX/k} = \frac{X^2 - jRX/k}{R + jX(k - 1/k)} = X\left(\frac{X - jR/k}{R + jX(k - 1/k)}\right),$$

or

$$z_k = X\left(\frac{\frac{X}{R} - j\frac{1}{k}}{1 + j\frac{X}{R}(k - 1/k)}\right).$$

We want $|z_k/z_o|$ = 1/10 when k = 54 MHz/50MHz = 1.08.

If we define B ≡ X/R, we have

$$\left|\frac{z_k}{z_o}\right| = \frac{\left|\dfrac{B - j\dfrac{1}{k}}{1 + jB(k - \dfrac{1}{k})}\right|}{|B - j|} = \frac{1}{10}.$$

At this point, it is prudent to attempt an approximation. Because $k \approx 1$, we may approximate B – j/*k* as B – j and cancel terms to obtain a simpler equation:

$$\left|\frac{z_k}{z_o}\right| \approx \frac{1}{\left|1 + jB(k - \dfrac{1}{k})\right|} = \frac{1}{10}.$$

Inverting both sides and applying the definition of magnitude, we have

$$\left|1 + jB(k - \frac{1}{k})\right| = \sqrt{1^2 + \left[B(k - \frac{1}{k})\right]^2} = 10.$$

Solving for B, we have

$$\left[B(k - \frac{1}{k})\right]^2 = 99 \quad \text{or} \quad B = \frac{\sqrt{99}}{k - \dfrac{1}{k}} = \frac{\sqrt{99}}{1.08 - \dfrac{1}{1.08}} \approx 64.58.$$

Using R = X/B = 31.8 Ω/64.58, we have R = 0.492 Ω.

At 54 MHz, we have

$$|z_k| = X\frac{\left|B - j\dfrac{1}{k}\right|}{|1 + jB(k - 1/k)|} = X\frac{\left|B - j\dfrac{1}{k}\right|}{10} \approx X\frac{|B|}{10} = 31.8\Omega\frac{64.58}{10}$$

We obtain $|z_k| \approx 205\ \Omega$ at 54 MHz.