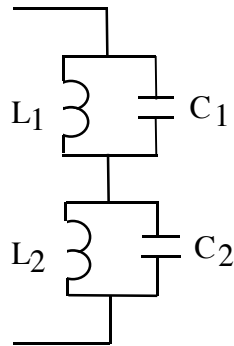


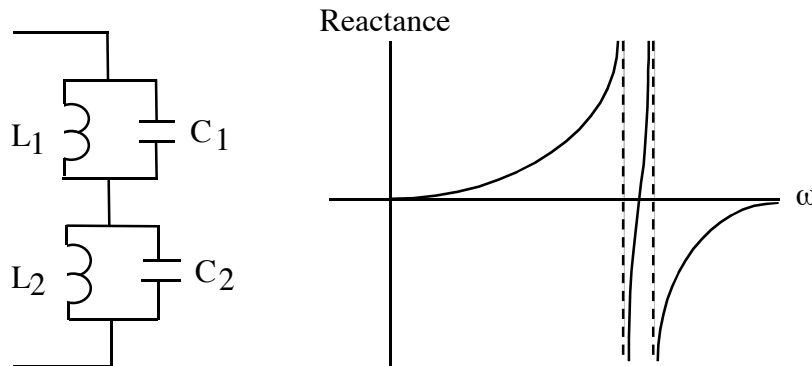
**EX:** In the circuit below,  $L_1C_1 = 0.8 L_2C_2$ . Draw reactance curves for this configuration to illustrate the location of all of the series and parallel resonant frequencies. Does the total circuit appear to be inductive or capacitive at very low frequencies? Does the total circuit appear to be inductive or capacitive at frequencies well above both of the parallel resonant frequencies?



**ANS:** The general shape of the curve is shown below. The highest and lowest resonant frequencies, indicated by dashed lines, are at frequencies whose ratio is  $1/\sqrt{0.8} \approx 0.9$ . There is also a resonance where the reactance passes through zero. The frequency of this resonance is

$$\omega_{12} = \sqrt{\frac{1}{\frac{(1+0.8)}{2} L_2 C_2}} = \omega_2 \sqrt{\frac{2}{1+0.8}}$$

At low frequency, the circuit looks inductive. At high frequency, the circuit looks capacitive.



**SOL'N:** A parallel L and C looks like an open circuit at resonance. If either of the L parallel C circuits looks like an open circuit, the entire circuit looks like an open circuit. Thus, we have resonant frequencies  $\omega_1$  and  $\omega_2$ :

$$\omega_1 = \frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{0.8 L_2 C_2}} = \frac{\omega_2}{\sqrt{0.8}}$$

$$\omega_2 = \frac{1}{\sqrt{L_2 C_2}}$$

At an intermediate frequency, the  $L_1$  and  $C_1$  circuit still looks inductive and the  $L_2$  and  $C_2$  circuit looks capacitive. At some frequency, the impedances exactly cancel. We can calculate that frequency in terms of resonant frequencies. We begin by writing the equation for the total impedance:

$$z = \frac{j\omega L_1 \cdot \frac{1}{j\omega C_1}}{j\omega L_1 + \frac{1}{j\omega C_1}} + \frac{j\omega L_2 \cdot \frac{1}{j\omega C_2}}{j\omega L_2 + \frac{1}{j\omega C_2}}$$

Dividing the top and bottom of each term by L gives an expression involving only resonant frequencies:

$$z = \frac{j\omega \cdot \frac{1}{j\omega L_1 C_1}}{j\omega + \frac{1}{j\omega L_1 C_1}} + \frac{j\omega \cdot \frac{1}{j\omega L_2 C_2}}{j\omega + \frac{1}{j\omega L_2 C_2}} = \frac{\frac{1}{L_1 C_1}}{j\omega + \frac{1}{j\omega L_1 C_1}} + \frac{\frac{1}{L_2 C_2}}{j\omega + \frac{1}{j\omega L_2 C_2}}$$

$$z = \frac{\omega_1^2}{j\omega + \frac{\omega_1^2}{j\omega}} + \frac{\omega_2^2}{j\omega + \frac{\omega_2^2}{j\omega}} = \frac{\omega_1^2}{j\omega - \frac{j\omega_1^2}{\omega}} + \frac{\omega_2^2}{j\omega - \frac{j\omega_2^2}{\omega}}$$

Now we rearrange and solve for  $z = 0$ :

$$\frac{\omega_1^2}{j\omega - \frac{j\omega_1^2}{\omega}} = -\frac{\omega_2^2}{j\omega - \frac{j\omega_2^2}{\omega}}$$

We invert both sides, divide by  $j$ , and multiply by  $\omega$ :

$$\frac{\omega^2 - \omega_1^2}{\omega_1^2} = -\frac{\omega^2 - \omega_2^2}{\omega_2^2}$$

Put both terms containing  $\omega$  on the left:

$$\omega^2 \left( \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} \right) = 2$$

Replacing  $\omega_1$  with its value in terms of  $\omega_2$  yields our final result:

$$\omega = \sqrt{\frac{2}{\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}}} = \sqrt{\frac{2}{\frac{0.8}{\omega_2^2} + \frac{1}{\omega_2^2}}} = \omega_2 \sqrt{\frac{2}{1 + 0.8}} \equiv \omega_{12}$$

**CHECK:** The frequency must be between  $\omega_1$  and  $\omega_2$ . Because the factor multiplying  $\omega_2$  is the square root of one over the average of the squared resonant frequencies, its value is between  $\omega_1$  and  $\omega_2$ . ✓

**CHECK:** The frequency must be symmetrical in  $\omega_1$  and  $\omega_2$ . From the first expression of the last equation above, we see that  $\omega_{12}$  is symmetrical in  $\omega_1$  and  $\omega_2$ . ✓