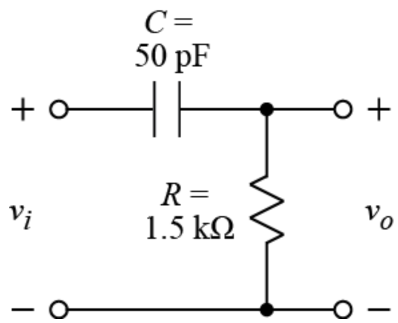
Ex:



$$v_i(t) = -4 + 2\sum_{k=1}^{\infty} \frac{1}{k^2+1} \sin(k\omega_0 t) \text{ V}$$

Write the time-domain expression of the third harmonic (i.e., $k = 3$) of $v_o(t)$.

Note: $\omega_o = \frac{10}{3}$M r/s for the Fourier series.

SOL'N: The third harmonic of $v_o(t)$, denoted here by $v_{o3}(t)$, is the response of the circuit to the third harmonic of the input signal,

$$v_{i3}(t) = 2 \cdot \frac{1}{k^2+1} \sin(k\omega_o t) \Big|_{k=3} \text{ V}.$$

or

$$v_{i3}(t) = \frac{2}{10} \sin(3\omega_o t) \text{ V} = \frac{1}{5} \sin\left(3 \cdot \frac{10}{3} \text{M} t\right) \text{V}$$

or

$$v_{i3}(t) = \frac{1}{5} \sin(10\text{M}t) \text{ V}$$

We convert $v_{i3}(t)$ to a phasor, V_{i3}, and compute impedance values for $\omega = 3\omega_o = 10$M r/s.

$$V_{i3} = -j\frac{1}{5}$$

$$\frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{-j}{10\text{M} \cdot 50\text{p}} \Omega = -j \, 2\text{k}\,\Omega$$

The output phasor, V_{o3}, is found using a voltage-divider formula.

$$V_{o3} = V_{i3} \cdot \frac{R}{R + \frac{1}{j\omega C}} = -j\frac{1}{5} \frac{1.5\cancel{K}}{1.5\cancel{K} - j2\cancel{K}} \text{ V}$$

$$= -j\frac{1}{5}\frac{3}{3-j4} \text{ V} = -j\frac{1}{5}\frac{3}{3-j4}\frac{3+j4}{3+j4} \text{ V}$$

$$= -j\frac{1}{5}\frac{3(3+j4)}{25} \text{ V} = \frac{1}{125}(12-j9) \text{ V}$$

$$= \frac{8(12-j9)}{8(125)} \text{ V} = 8(12-j9) \text{ mV}$$

$$V_{o3} = 96 - j72 \text{ mV}$$

Thus, $v_{o3}(t) = 96\text{ mV} \cos(10\text{M}t) + 72\text{ mV} \sin(10\text{M}t)$

or $v_{o3}(t) = 120\text{ mV} \cos(10\text{M}t - 37^\circ)$