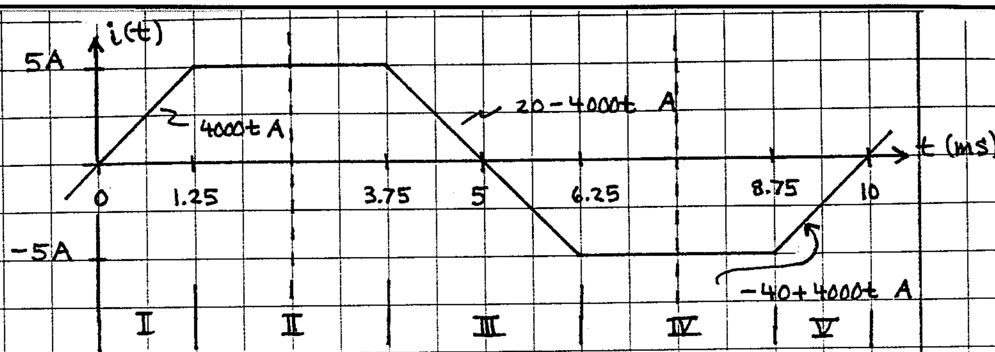


a) Fundamental frequency, f_0, in Hertz = ?
b) Is function even?
c) Is function odd?
d) Does function have half-wave (shift flip) symmetry?
e) " " " quarter-wave symmetry?
f) Find a_ν, a_k, b_k.

ans: a) 100 Hz
b) No.
c) Yes
d) Yes
e) Yes
f) $a_\nu = 0$ $a_k = 0$ all k $b_k = \begin{cases} 0 & k \text{ even} \\ \dfrac{80}{\pi^2 k^2} \sin \dfrac{k\pi}{4} & k \text{ odd} \end{cases}$

sol'n: a) $f_0 = \dfrac{1}{T} = \dfrac{1}{10 \text{ ms (period of waveform)}} = 100 \text{ Hz}$

b) Function not mirror image around vertical axis ⇒ not even
i.e. $f(t) \neq f(-t)$.

c) Function is flipped mirror image around vertical axis ⇒ odd
i.e. $f(t) = -f(-t)$.

d) If we shift one-half cycle and flip vertically, (i.e. mult by -1), then we get same waveform ⇒ half-wave symmetry
i.e. $-f(t - T/2) = f(t)$.

e) Mirror image around quarter wave points, (vertical dashed lines) ⇒ quarter wave symmetry
i.e. $f\left(\frac{T}{4} - t\right) = f\left(\frac{T}{4} + t\right)$ and $f\left(\frac{3T}{4} - t\right) = f\left(\frac{3T}{4} + t\right)$.

f) func odd ⇒ equal area above and below zero ⇒ $a_\nu = 0$, (ave = 0).
func odd ⇒ cos term coeff's = 0 since cos()'s are even funcs.
$\therefore$ $a_k = 0$ all k

half wave (shift flip) symmetry $\Rightarrow$ even harmonics line up with fund in such a way that product $i(t)\cos k\omega_0 t$ or $i(t)\sin k\omega_0 t$ is waveform with equal positive and negative areas $\Rightarrow a_{k\text{ even}} = 0$ and $b_{k\text{ even}} = 0$.

So we are left with only $b_{k\text{ odd}}$ terms.

$$b_{k\text{ odd}} = \frac{2}{T}\int_0^T i(t)\sin(k\omega_0 t)\,dt$$

Because $i(t)$ has different functional forms on successive intervals, (labeled I–V on plot of $i(t)$), we break our integral into 5 pieces.

Intervals: $\mathrm{I} = \left[0, \frac{T}{8} = 1.25\text{ ms}\right]$

$\mathrm{II} = \left[\frac{T}{8} = 1.25\text{ ms}, \frac{3T}{8} = 3.75\text{ ms}\right]$

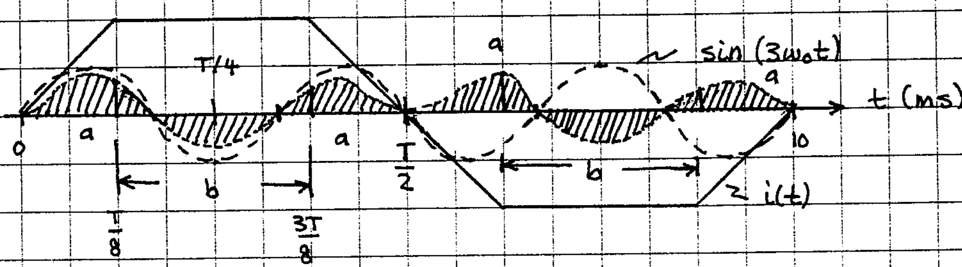
$\mathrm{III} = \left[\frac{3T}{8} = 3.75\text{ ms}, \frac{5T}{8} = 6.25\text{ ms}\right]$

$\mathrm{IV} = \left[\frac{5T}{8} = 6.25\text{ ms}, \frac{7T}{8} = 8.75\text{ ms}\right]$

$\mathrm{V} = \left[\frac{7T}{8} = 8.75\text{ ms}, T = 10\text{ ms}\right]$

$$b_{k\text{ odd}} = \frac{2}{T}\left[\begin{array}{ll} \displaystyle\int_0^{T/8} i(t)\sin(k\omega_0 t)\,dt & i(t) = 4000t\text{ A} \\ \displaystyle + \int_{T/8}^{3T/8} i(t)\sin(k\omega_0 t)\,dt & i(t) = 5\text{A} \\ \displaystyle + \int_{3T/8}^{5T/8} i(t)\sin(k\omega_0 t)\,dt & i(t) = 20 - 4000t\text{ A} \\ \displaystyle + \int_{5T/8}^{7T/8} i(t)\sin(k\omega_0 t)\,dt & i(t) = -5\text{A} \\ \displaystyle + \int_{7T/8}^{T} i(t)\sin(k\omega_0 t)\,dt & i(t) = -40 + 4000t\text{ A} \end{array}\right]$$

Now we exploit symmetries.



Above, we have a plot of $i(t)$, $\sin(3\omega_0 t)$, and $i(t)\cdot\sin(3\omega_0 t)$ [shown cross-hatched].

We observe that the four areas labeled 'a' have the same shape (though some are time reversed) and same area (even if they are time reversed).

Note: $\int_0^d f(t)\,dt = \int_{-d}^0 f(-t)\,dt$; same area if time reversed.

Similarly, the two areas labeled 'b' have the same shape and area.

Thus, $b_{k\,\text{odd}} = \frac{2}{T}\cdot\left[4\cdot\text{area 'a'} + 2\cdot\text{area 'b'}\right]$

We can go further. 'b' is symmetric about the quarter-wave points. $\therefore$ area 'b' $= 2\cdot$ area (half of 'b')

$$\therefore\ b_{k\,\text{odd}} = \frac{2}{T}\left[4\ \text{area 'a'} + 4\cdot\text{area half of 'b'}\right]$$

$$= \frac{2}{T}\left[4\int_0^{T/8} i(t)\sin(k\omega_0 t)\,dt + 4\int_{\frac{T}{8}}^{T/4} i(t)\sin(k\omega_0 t)\,dt\right]$$

move outside move outside

$$= \frac{8}{T}\left[\int_0^{T/8} 4000\,t\sin(k\omega_0 t)\,dt + \int_{T/8}^{T/4} 5A\sin(k\omega_0 t)\,dt\right]$$

from Text p. 1009 (integral table) we have:

$$\int x\sin ax\,dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax$$

$$\therefore\ b_{k\,\text{odd}} = \frac{8}{T}\left[4000\left\{\frac{1}{(k\omega_0)^2}\sin k\omega_0 t - \frac{t}{k\omega_0}\cos k\omega_0 t\right\}\Bigg|_0^{T/8} - 5A\,\frac{\cos k\omega_0 t}{k\omega_0}\Bigg|_{T/8}^{T/4}\right]$$

Use $\omega_0 = 2\pi/T$

$$b_{k\,odd} = \frac{8}{T}\left(4000\left\{\frac{1}{(k\omega_0)^2}\left[\sin k\frac{2\pi}{\cancel{T}}\frac{\cancel{T}}{8} - \underbrace{\sin 0}_{0}\right]\right.\right.$$

$$\left.- \frac{1}{k\omega_0}\left[\frac{T}{8}\cos k\frac{2\pi}{\cancel{T}}\frac{\cancel{T}}{8} - \underbrace{0\cdot\cos 0}_{0}\right]\right\}$$

$$\left.-5\frac{1}{k\omega_0}\left[\cos k\frac{2\pi}{\cancel{T}}\frac{\cancel{T}}{4} - \cos k\frac{2\pi}{\cancel{T}}\frac{\cancel{T}}{8}\right]\right)$$

$$= \frac{8}{T}\left(\frac{4000}{k^2\frac{(2\pi)^2}{T^2}}\sin k\frac{\pi}{4} - \frac{4000}{k\frac{2\pi}{T}}\frac{T}{8}\cos k\frac{\pi}{4}\right.$$

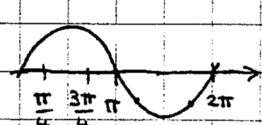
$$\left.-\frac{5}{k\frac{2\pi}{T}}\underbrace{\cos k\frac{\pi}{2}}_{0 \text{ for } k \text{ odd}} + \frac{5}{k\frac{2\pi}{T}}\cos k\frac{\pi}{4}\right)$$

We observe that $4000\cdot\frac{T}{8} = 500\cdot T = 500\cdot 10\,ms = 5$

$\therefore$ The cos terms cancel.

$$b_{k\,odd} = \frac{8}{T}\frac{4000}{k^2\frac{(2\pi)^2}{T^2}}\sin\frac{k\pi}{4} = \left(\frac{8\cdot\cancel{4000}^{1000}\cdot T^{\cancel{2}}}{\cancel{T}\cdot k^2\cdot\cancel{4}\pi^2} = \frac{8000\,T}{k^2\pi^2}\right)\cdot\sin\frac{k\pi}{4}$$

$$= \frac{8000\cdot 10\,ms}{k^2\pi^2}\sin\frac{k\pi}{4} = \frac{80}{k^2\pi^2}\sin\frac{k\pi}{4}\ A$$



Note: $\sin\frac{k\pi}{4} = \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, \ldots$

$k = 1, 3, 5, 7, \ldots$

So $$i(t) = \sum_{\substack{k\,odd\\k>0}}^{\infty}\frac{80}{k^2\pi^2}\sin\left(\frac{k\pi}{4}\right)\sin(k\omega_0 t) \qquad \omega_0 = \frac{2\pi}{10\,ms} = 2\pi\cdot 100\text{ rad/s}$$