tool: We find the normalizing constant for the N-dimensional Fourier series basis functions by using the simplest cos term.

$$\text{Basis func} = \alpha \cos(2\pi \vec{n} \circ \vec{x}) \quad \text{or} \quad \alpha \sin(2\pi \vec{n} \circ \vec{x})$$

The simplest term is
$$\alpha \cos 2\pi(1,0,\dots,0) \circ (x_1,\dots,x_N)$$
$$= \alpha \cos 2\pi x_1$$

The inner product of the basis function with itself must have value 1.

$$|\alpha \cos 2\pi x_1|^2 = (\alpha \cos 2\pi x_1, \alpha \cos 2\pi x_1) = 1 \quad \text{required}$$

tool: The inner product for N-dimensional Fourier coefficients of $f(\vec{x})$ is an N-dimensional integral. Domain $= [\frac{1}{2}, \frac{1}{2}]^N$

$$a_{\vec{n}} = (f(\vec{x}), \alpha \cos 2\pi \vec{n} \circ \vec{x}) = \int_{-1/2}^{1/2} \dots \int_{-1/2}^{1/2} f(\vec{x})\, \alpha \cos 2\pi \vec{n} \circ \vec{x} \; dx_1 \dots dx_N$$

We use this inner product formula to now find the value of the normalizing constant α.

$$1 = (\alpha \cos 2\pi x_1, \alpha \cos 2\pi x_1) = \int_{-1/2}^{1/2} \dots \int_{-1/2}^{1/2} \alpha^2 \cos^2 2\pi x_1 \; dx_1 \dots dx_N$$

ave of $\cos^2 = \frac{1}{2}$
$$= \int_{-1/2}^{1/2} \dots \int_{-1/2}^{1/2} \alpha^2 \cdot \frac{1}{2} \cdot \left(\frac{1}{2} - -\frac{1}{2}\right) dx_2 \dots dx_N$$

$$= \frac{\alpha^2}{2} \int_{-1/2}^{1/2} \dots \int_{-1/2}^{1/2} dx_2 \dots dx_N$$

$$= \frac{\alpha^2}{2} \cdot 1$$

$$\therefore \alpha^2 = 2 \quad \text{or} \quad \alpha = \sqrt{2}$$