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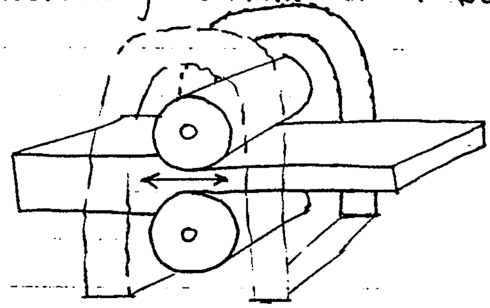
case study: The problem of approximating a functions of several ~~many~~ variables arises in many industrial applications. Here we consider the problem of controlling a mill in a steel plant.



The reversing mill shown here rolls a slab of hot steel to reduce its thickness. The slab entering the mill is 8.6" thick. It leaves the mill at 1.0" thick. It rolls through the mill in the forward and then the reverse direction for a total of five passes.

To control this mill we must choose the gap between the rolls to give us the desired thickness (also called gauge) after each pass. We don't have to change the roll gap once the bar is in the mill. Thus, we have a static setup problem. Choosing the roll gap involves the function approximations described below. We also consider temperature and width.

ex: To set the roll gap, we must not exceed the maximum allowable vertical force on the mill. Thus, we must model the force. The following variables are inputs to the model:

1) R = roll radius
2) h_{out} = gauge when bar leaves mill
3) Δh = draft (change in gauge when bar is rolled)
4) μ_s = friction coefficient for bar-roll contact
5) ω = motor speed
6) T = bar temperature ($\approx 2000\,°F$)
7) χ = bar hardness (depends on steel chemistry)

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We have a function of seven variables:

$$\text{force} = f(R, h_{out}, \Delta h, \mu_s, \omega, T, \chi)$$

In practice it difficult to gather enough data to use a curve fit approximation to a function of seven variables. Instead, we rely on physics and tunable coefficients. Nevertheless, we often encounter sub equations in the model to which we can apply generic function methods.

ex: Hardness χ in the force model is itself a function of variables relating to steel chemistry:

1) % Carbon
2) % Silicon
3) % Vanadium
4) % Nitrogen
5) % Phosphorus

⋮

The standard approach is to use multiple regression to model χ:

$$\chi \approx \%C + \frac{\%\,Si}{5} + 3\,\%\,Va + 1.5\,N + \frac{P}{10} + \ldots$$

This formula (which is not the actual one used but is similar to the actual one) gives a resistance factor value in the range 1–100 with $\chi = 25$ being typical.

The problem with this χ formula is that it is linear whereas χ is actually a nonlinear function of chemistry. Since the χ formula is derived empirically from data measurements, this is an ideal application for curve fitting function approximation methods.

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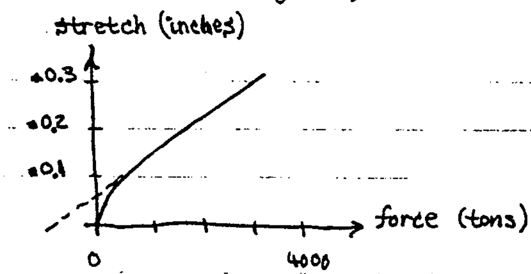
ex: When we set the roll gap in the mill, we must also consider *mill stretch*: the force of the bar pushing back on the rolls causes the mill housing — massive though it is — to stretch like a spring. The mill stretch is almost strictly a function of force, f, and is described by a spring equation, (Hook's law):

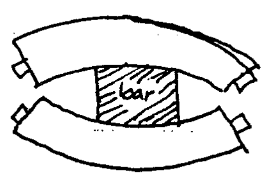


$$\text{stretch} = \delta h_s(f)$$

Note that the stretch is nonlinear near the origin. Otherwise, it is quite linear. Thus, a piecewise approximation is suitable as a model for mill stretch. A piecewise linear approximation corresponds to triangular radial basis functions in one dimension.

To compensate for mill stretch we reduce the roll gap by δh_s.

ex: The force of rolling also bends the mill rolls, especially for narrow bars.



This roll deflection, though slight, is noticeable. The deflection is a function of force and bar width:

$$\text{deflection} = \delta h_d(f, w)$$

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The standard formula for deflection is linear in the width:

$$\delta h_d \approx k_d \frac{w_{max} - w}{w_{max}} f$$

where k_d = constant (gives $\delta h_d \approx .01$ for 1000 tons force and $w = w_{max}/2$)

w_{max} = 132" maximum bar width
f = force in tons for rolling bar
w = actual bar width

This formula is hard to improve on because the deflection is small anyway. Thus, a simple approximation is cost effective.

ex: When we roll a bar it gets longer and wider. The change in width is particularly difficult to model. The width change is a function of the same variables that affect force:

$$\text{width change} = \Delta w(R, h_{out}, \Delta h, \mu_s, \omega, T, \chi)$$

The most successful modeling of Δw to date relies on lookup tables. Thus, width control is a prime candidate for function approximation methods – especially since Δw is a function of so many variables.

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Bar heating and cooling occurs in the following ways:

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1) Water sprays cool the bar
2) The bar radiates heat
3) Furnaces heat the bars before rolling
4) Contact with mill rolls cools the bar
5) Deforming the bar ~~during~~ by rolling heats the bar

Good physical models exist for calculating the temperature of the bar when the rolling procedure is known. If we wish to control the temperature, however, we must invert the models. In many cases the inverse can be found only by an iterative calculation. If we perform that iterative calculation ahead of time we can store the results in a table and interpolate between points. Function approximation methods are suitable for this task.

An example of a function we might wish to compute is the holding time (between passes) that will yield a desired temperature. The bar's change in temperature for a pass is a function of the following variables:

1) T_{in} = temperature of bar at mill entry
2) ω = mill speed
3) h_{in} = entry gauge of bar
4) h_{out} = exit " " "
5) w = amount of water sprayed on bar

$$\text{change in temperature} = \Delta T\,(T_{in}, \omega, h_{in}, h_{out}, w)$$

Given the ΔT we could use a heat radiation model to find the holding time, t_{hold}. On the other hand, we could create an inverse model that gives holding time directly from the input variables:

$$\text{hold time} = t_{hold}\,(T_{in}, \omega, h_{in}, h_{out}, w)$$