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$\mathcal{G}$ is a <u>complete set of functions</u> in space $\mathcal{F} \equiv$

the closure of the set of all linear combinations of functions from $\mathcal{G}$ is $\mathcal{F}$.

i.e. $\mathcal{F} = \bar{\mathcal{L}}(\mathcal{G})$

ex: $\mathcal{G} = \{ \cos 2\pi n x, \sin 2\pi n x : n = 0, \ldots, \infty \}$ is a complete set of functions for $\mathcal{F}$ = all periodic continuous functions on the unit ~~square~~ interval $[0,1]$.

In this case the linear combinations of functions from $\mathcal{G}$ are Fourier series:

$$\sum_{n=0}^{\infty} a_n \cos 2\pi n x + b_n \sin 2\pi n x \qquad a_n, b_n \in \mathbb{R}$$

We know that we can represent any function in $\mathcal{F}$ = periodic continuous functions on $[0,1]$ with such a Fourier series.

note: The linear combinations of functions from $\mathcal{G}$ are functions of form $\sum_{n=0}^{\infty} a_n g_n(x)$ where $g_n(x)$ is in $\mathcal{G}$.
These linear combinations form a linear space $\mathcal{F}$ in that if $f_1 \in \mathcal{F}$ and $f_2 \in \mathcal{F}$ then $a_1 f_1 + a_2 f_2 \in \mathcal{F}$.