Neil E Cotter
30 Jan 1994
Neil E Cotter
2 Feb 1994

$\mathcal{F}$ is a <u>linear space</u> over $\mathcal{A}$ $\equiv$ given a_1, a_2 constants $\in \mathcal{A}$
given $f_1, f_2 \in \mathcal{F}$
OR $\mathcal{F}$ is a <u>vector space</u> over $\mathcal{A}$ (same thing) then $a_1 f_1 + a_2 f_2 \in \mathcal{F}$

ex: Vectors $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ with $x_1, x_2 \in \mathbb{R}$ form a vector space over $\mathbb{R}$ = real numbers.

pf: $a_1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + a_2 \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} a_1 x_1 + a_2 x_3 \\ a_1 x_2 + a_2 x_4 \end{bmatrix}$

If a_1, a_2, x_1, x_2, x_3, and x_4 are real then so ~~is~~ are $a_1 x_1 + a_2 x_3$ and $a_1 x_2 + a_2 x_4$. Thus, the linear combination of vectors in the space is also in the space.

We call this space $\mathbb{R}^2$.

note: In formal terms, $\mathcal{A}$ must be a field. $\mathcal{A}$ contains a zero element. $\mathcal{F}$ may be a set of elements, such as vectors, or it may be a set of ~~as~~ functions.

ex: ~~All~~ $C(-\infty, \infty) \equiv$ continuous real-valued functions on the interval $(-\infty, \infty)$ forms a linear space over $\mathbb{R}$.

pf: given real-valued continuous functions $f_1, f_2 \in C(-\infty, \infty)$ and real numbers a_1, a_2, then $a_1 f_1 + a_2 f_2$ will also be real-valued and continuous. (The sum of continuous functions is continuous).

ex: The set of functions $\mathcal{F} = \{ e^{rx} : r \in \mathbb{R} \}$ does not ~~forms~~ form a linear space over $\mathbb{R}$

pf: $a_1 e^{r_1 x} + a_2 e^{r_2 x}$ <u>cannot</u> always be written in the form $a e^{rx} \in \mathcal{F}$.