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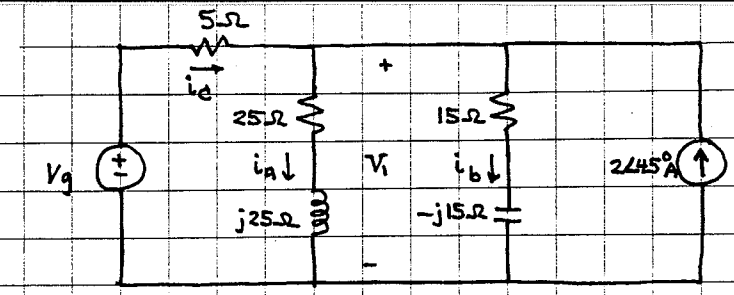
Ref: Nilsson & Riedel, Rev 6, Prob 9.33

IMPEDANCE CIRCUITS
KIRCHHOFF'S LAWS
Example

ex:



Given $\mathbb{I}_b = 5\angle 45°$ A is phasor for current i_b.

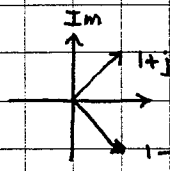
a) Find: $\mathbb{I}_a$, $\mathbb{I}_c$, and V_g phasors.

sol'n: Define phasor voltage V_1 as shown.

$$\mathbb{V}_1 = \mathbb{I}_b \cdot (15\Omega - j15\Omega) \qquad \text{Ohm's Law}$$

and $$\mathbb{V}_1 = \mathbb{I}_a \cdot (25\Omega + j25\Omega) \qquad " \quad " \quad \text{same } \mathbb{V}$$

$$\therefore \mathbb{I}_a = \mathbb{I}_b \frac{15\Omega - j15\Omega}{25\Omega + j25\Omega} = \mathbb{I}_b \frac{15}{25} \frac{1-j}{1+j}$$



$$1-j = \sqrt{2}\, e^{-j\pi/4} \qquad 1+j = \sqrt{2}\, e^{j\pi/4}$$

$$\frac{1-j}{1+j} = \frac{\sqrt{2}\, e^{-j\pi/4}}{\sqrt{2}\, e^{j\pi/4}} = e^{-j\pi/2} = -j$$

$$\therefore \mathbb{I}_a = \mathbb{I}_b \frac{15}{25}(-j) \qquad -j = 1\angle -90°$$

$$= 5\angle 45° \cdot \frac{3}{5} \cdot 1\angle -90° = 5 \cdot \frac{3}{5} \cdot 1 \angle 45° - 90°$$

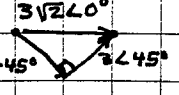
$$\mathbb{I}_a = 3\angle -45° \text{ A}$$

Now sum currents for V_1 node:

$$\mathbb{I}_c = \mathbb{I}_a + \mathbb{I}_b - 2\angle 45° \text{ A} = 3\angle -45° + \underbrace{5\angle 45° - 2\angle 45°}_{\text{same angle, can add magnitudes}} \text{ A}$$

$$= 3\angle -45° + 3\angle -45° \text{ A}$$

$$= 3\sqrt{2}\angle 0° \text{ A}$$



$$\mathbb{V}_g = \mathbb{I}_c \cdot 5\Omega + \mathbb{I}_b (15 - j15\Omega) = 3\sqrt{2} \cdot 5\Omega + \underbrace{5\angle 45° \cdot 15\sqrt{2}\angle -45°}_{5 \cdot 15\sqrt{2}\angle 45° - 45°} \text{ V}$$

$$= 15\sqrt{2} \text{ V} + 75\sqrt{2} \text{ V} = 90\sqrt{2} \text{ V}$$

b) If $\omega = 800$ rad/s, write expressions for $i_a(t)$, $i_c(t)$, $v_g(t)$.

Soln: $\mathbf{I}_a = 3\angle -45^\circ$ A $\qquad -45^\circ = -\frac{\pi}{4}$ rad

$\therefore i_a(t) = 3\cos\left(\omega t - \frac{\pi}{4}\right)$

$\mathbf{I}_c = 3\sqrt{2}\angle 0^\circ$ A

$\therefore i_c(t) = 3\sqrt{2}\cos(\omega t)$

$\mathbf{V}_g = 90\sqrt{2}$ V $\quad$ or $\quad 90\sqrt{2}\angle 0^\circ$ V

$\therefore v_g(t) = 90\sqrt{2}\cos(\omega t)$