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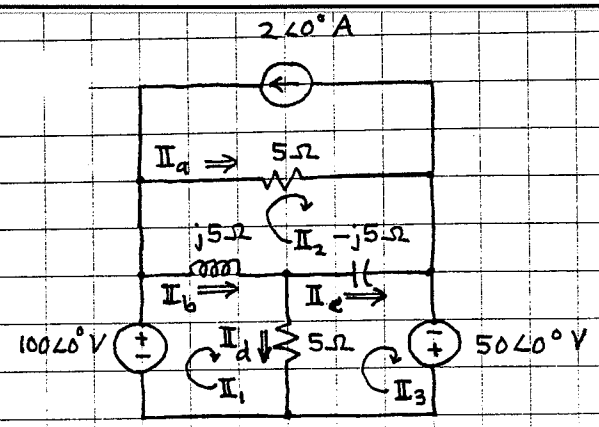
Ref: Nilsson & Riedel, Rev 6, Prob 9.52

IMPEDANCE CIRCUITS
MESH-CURRENT METHOD
Example

ex:



Use mesh-current method to find $\mathbb{I}_a, \mathbb{I}_b, \mathbb{I}_c, \mathbb{I}_d$.

sol'n: Define mesh currents (phasors) as indicated on above diagram. Note that the top loop lacks a mesh current because that loop must have current phasor equal to the $2\angle 0°A$ of the current source on the outside edge of the loop. Note also that we could choose any mesh loop to be $\mathbb{I}_1$. The solution manual chose the lower left loop to be $\mathbb{I}_1$.

Now we write equations as we would for a DC circuit, but we use complex impedances instead of R's and phasor current and voltage sources instead of real-valued current and voltage sources.

Loop 1: $100\angle 0°V - \mathbb{I}_1\, j5\Omega - \mathbb{I}_1\, 5\Omega = 0V$
$+ \mathbb{I}_2(j5\Omega) + \mathbb{I}_3\, 5\Omega$

Loop 2: $-\mathbb{I}_2\, j5\Omega - \mathbb{I}_2\, 5\Omega - \mathbb{I}_2(-j5\Omega) = 0V$
$+ \mathbb{I}_1\, j5\Omega - 2\angle 0°A \cdot 5\Omega + \mathbb{I}_3(-j5\Omega)$

Loop 3: $50\angle 0°V - \mathbb{I}_3\, 5\Omega - \mathbb{I}_3(-j5\Omega) = 0V$
$+ \mathbb{I}_1\, 5\Omega + \mathbb{I}_2(-j5\Omega)$

Now simplify. Put constants on right sides of eq'ns, and write left sides as:
$\mathbb{I}_1 \cdot$ coeff $+ \mathbb{I}_2 \cdot$ coeff $+ \mathbb{I}_3 \cdot$ coeff.

Loop 1: $\mathbb{I}_1(5+j5)\,\Omega - \mathbb{I}_2\, j5\,\Omega + \mathbb{I}_3(-5)\,\Omega = 100\angle 0^\circ\ V$

Loop 2: $\mathbb{I}_1(-j5)\,\Omega + \mathbb{I}_2\,5\,\Omega + \mathbb{I}_3\, j5\,\Omega = -2\angle 0^\circ A \cdot 5\,\Omega = -10\angle 0^\circ\ V$

Loop 3: $\mathbb{I}_1(-5)\,\Omega + \mathbb{I}_2\, j5\,\Omega + \mathbb{I}_3(5-j5)\,\Omega = 50\angle 0^\circ\ V$

Eliminate $\mathbb{I}_3$: eqn 1 + eqn 2 + eqn 3 =
(and by good fortune $\mathbb{I}_1$, too)

$$\mathbb{I}_1(5+j5-j5-5)\,\Omega + \mathbb{I}_2(-j5+5+j5)\,\Omega + \mathbb{I}_3(-5+j5+5-j5)\,\Omega$$

$$= 100\angle 0^\circ - 10\angle 0^\circ + 50\angle 0^\circ\ V$$

$$\mathbb{I}_2 = \frac{140\angle 0^\circ\ V}{5\,\Omega} = 28\ A$$

Now solve for $\mathbb{I}_1$: j eqn 1 + eqn 2 =

$$\mathbb{I}_1(-5+j5-j5)\,\Omega + \mathbb{I}_2\cdot(5+5)\,\Omega + \mathbb{I}_3(-j5+j5)\,\Omega = \underbrace{j100}_{5\angle 90^\circ}\ \underbrace{-10}_{10\angle 180^\circ}\angle 0^\circ\ V$$

or $\mathbb{I}_1(-5)\,\Omega + 28A\cdot 10\,\Omega = -10 + j100\ V$

or $\mathbb{I}_1(-5)\,\Omega = -290 + j100\ V$

or $\mathbb{I}_1 = 58 - j20\ A$

Now find $\mathbb{I}_3$ from eqn 2:

$$(58-j20)A\,(-j5)\,\Omega + 28A\cdot 5\,\Omega + \mathbb{I}_3\, j5\,\Omega = -10\ V$$

or $\mathbb{I}_3 = \dfrac{-10 + 100 + j290 + -j40\ V}{j5\,\Omega} = 58 + j10\ A$

$\mathbb{I}_a = \mathbb{I}_2 + 2\angle 0^\circ A = 28A + 2A = 30A$ $\quad \mathbb{I}_b = \mathbb{I}_1 - \mathbb{I}_2 = 58 - j20 - 28A = 30 - j20A$

$\mathbb{I}_c = \mathbb{I}_3 - \mathbb{I}_2 = 58 + j10 - 28A = 30 + j10A$ $\quad \mathbb{I}_d = \mathbb{I}_1 - \mathbb{I}_3 = 58 - j20 - 58 - j10A = -j30A$