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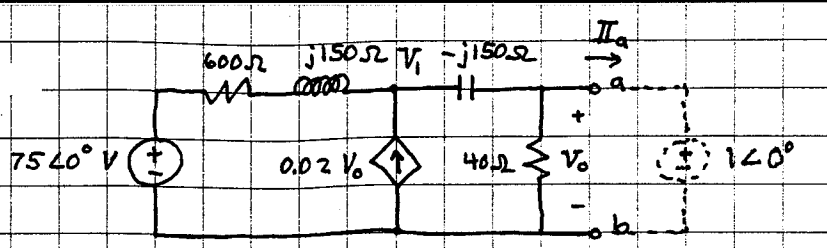
Ref: Nilsson & Riedel, Rev 6, Prob 9.43

IMPEDANCE CIRCUITS
THEVENIN EQUIVALENT
Example 2

ex:



Find Thevenin equivalent circuit with respect to a-b.

sol'n: Find V_{Th} as open circuit output V_o.
Find z_{Th} by turning independent sources to zero and connecting $1\angle 0°$ V source to output.
Then $z_{Th} = \dfrac{1\angle 0° \text{ V}}{I_a}$

$$V_{Th}: \quad \frac{V_1}{-j150\Omega + 40\Omega} - 0.02 V_o + \frac{V_1 - 75\angle 0° \text{V}}{600\Omega + j150\Omega} = 0\text{A}$$

$$V_o = V_1 \frac{40\Omega}{-j150\Omega + 40\Omega}$$

$$V_1 \left(\frac{1}{-j150\Omega + 40\Omega} - \frac{-0.8}{-j150\Omega + 40\Omega} + \frac{1}{600\Omega + j150\Omega} \right)$$

$$= \frac{75\angle 0° \text{V}}{600\Omega + j150\Omega}$$

$$V_1 \left(0.2 + \frac{40\Omega - j150\Omega}{600\Omega + j150\Omega} \right) = \frac{75 \text{ V} \ (40\Omega - j150\Omega)}{600\Omega + j150\Omega}$$

$$V_1 \ (120\Omega + j30\Omega + 40\Omega - j150\Omega) = 75 \text{V} (40\Omega - j150\Omega)$$

$$V_1 = \frac{75 \text{V} (4 - j15)}{16 - j12} = \frac{75 \text{ V} (4 - j15)}{4 (4 - j3)} = \frac{75\text{V} \sqrt{16+225}\, e^{j \tan^{-1} \frac{-15}{4}}}{4 \cdot 5\Omega \, e^{j \tan^{-1} \frac{-3}{4}}}$$

$$= 58.2\text{V} \ e^{j\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{-15}{4}\right)}$$

$$= 58.2 \angle \left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{-15}{4} \right)$$

$$V_o = \frac{40\Omega}{40\Omega - j150\Omega} V_1$$

$$= \frac{4}{4 - j15} V_1$$

$$= \frac{4.75\ V}{20\, e^{j\tan^{-1}-\frac{3}{4}}}$$

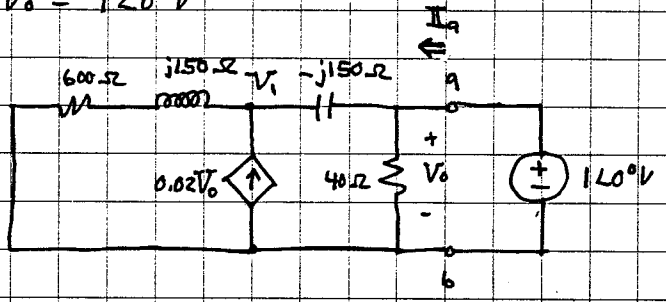
$$= 15 \angle \tan^{-1}\frac{3}{4}\ V$$

$$V_{Th} = V_o = 15 \angle 36.9^\circ\ V$$

$$Z_{Th} = \frac{1\angle 0^\circ V}{I_a}$$

Connect 1V source (sinusoidal) to a-b.
Turn independent source to 0.

$V_o = 1\angle 0^\circ V$



Use Node-v to find V_1. $0.02V_o = 0.02 \cdot 1\angle 0^\circ = 0.02A$.

$$\frac{V_1}{600\Omega + j150\Omega} - 0.02A + \frac{V_1 - 1\angle 0^\circ V}{-j150\Omega} = 0A$$

$$V_1\left(\frac{1}{600\Omega + j150\Omega} + \frac{1}{-j150\Omega}\right) = \frac{1\angle 0^\circ V}{-j150\Omega} + 0.02A$$

$$V_1\left(1 + \frac{600\Omega + j150\Omega}{-j150\Omega}\right) = \left(\frac{1\ V}{-j150\Omega} + 0.02A\right)(600\Omega + j150\Omega)$$

$$V_1\left(-j150\Omega + 600\Omega + j150\Omega\right) = (+1V - \underbrace{j0.02 \cdot 150}_{j3}\ V)(600\Omega + j150\Omega)$$

$$V_1 = (+1 - j3)(600\Omega + j150\Omega)/600\Omega$$

$$V_1 = (1-j3)\left(1+\frac{j}{4}\right) = 1+\frac{3}{4}+j\left(\frac{1}{4}-3\right) = \frac{7}{4}-j\frac{11}{4}$$

$$-I_a = \frac{V_1 - 1\angle 0^\circ\ V}{-j150\ \Omega} - \frac{1\angle 0^\circ\ V}{40\ \Omega}$$

$$= \frac{1}{10}\left(\frac{jV_1 - j}{15} - \frac{1}{4}\right)$$

$$= \frac{1}{10}\,\frac{1}{60}\left(j4V_1 - j4 - 15\right)$$

$$= \frac{1}{600}\left(j(7-j11) - j4 - 15\right)$$

$$= \frac{1}{600}\left(j3 - 4\right)$$

$$Z_{Th} = \frac{1\angle 0^\circ\ V}{I_a}$$

$$= \frac{-600}{j3-4}\ \Omega$$

$$= \frac{-600}{3^2+4^2}(-j3-4)$$

$$= +\frac{600}{25}(4+j3)$$

$$= +24\,(4+j3)$$

$$= +24\sqrt{4^2+3^2}\,e^{j\tan^{-1}\frac{3}{4}}$$

$$= +120\,e^{j36.87^\circ}$$