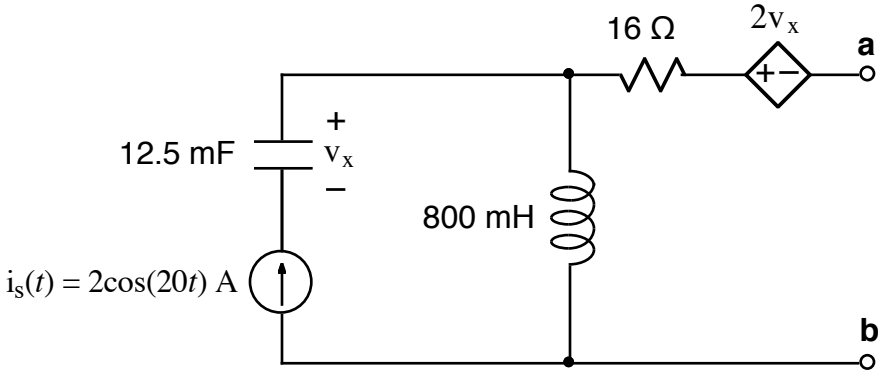
Ex:



a) Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for $i_s(t)$, and show numerical impedance values for R, L, and C. Label the dependent source appropriately.

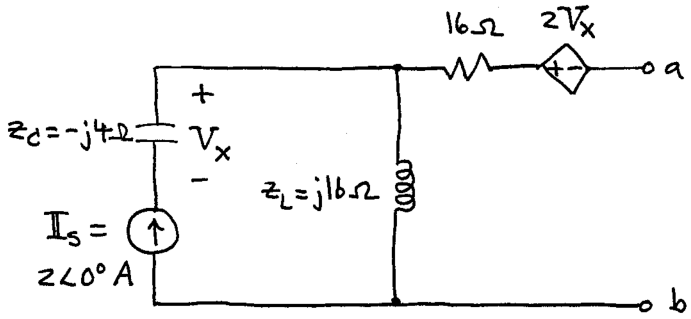
b) Find the Thevenin equivalent (in the frequency domain) for the above circuit. Give the numerical phasor value for $\mathbf{V}_{Th}$ and the numerical impedance value of z_{Th}.

Soln: a) $\omega = 20$ r/s from $i_s(t)$

$$\mathbb{I}_s = 2\angle 0^\circ \text{ A}$$

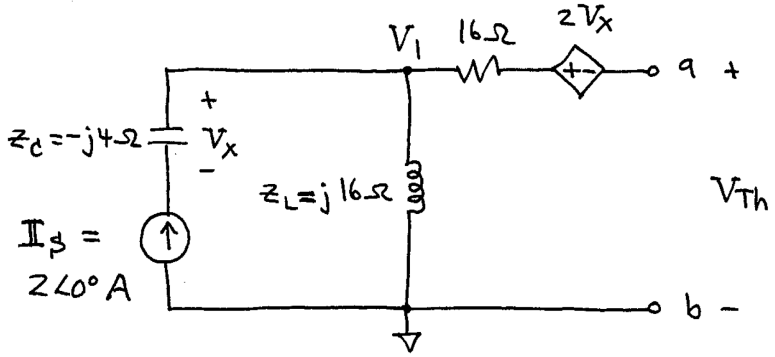
$$z_c = \frac{-j}{\omega C} = \frac{-j}{20\text{r/s} \cdot 12.5\text{mF}} = \frac{-j\ \Omega}{250\text{m}} = -j4\ \Omega$$

$$z_L = j\omega L = j\ 20\text{r/s} \cdot 800\text{mH} = j\ 16\text{K}\Omega = j16\ \Omega$$



b) $V_{Th} = V_{ab}$ open circuit.

Use node voltage V_1:



V_1 (from $\mathbb{I}_S \cdot z_L$) $= 2\angle 0°\,A \cdot j16\Omega = j32\,V$

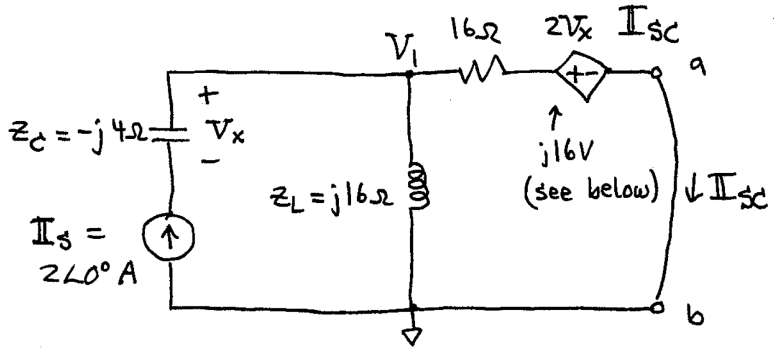
We've 0V across 16Ω since no current flows.

Thus, $V_{Th} = V_1 - 2V_x$.

$V_x = -\mathbb{I}_S z_C = -2\angle 0°\,A \cdot (-j4\Omega) = j8\,V$

So $V_{Th} = j32V - 2(j8V) = j16V$ or $16\angle -90°\,V$

To find z_{Th}, use $z_{Th} = \dfrac{V_{Th}}{\mathbb{I}_{SC}}$

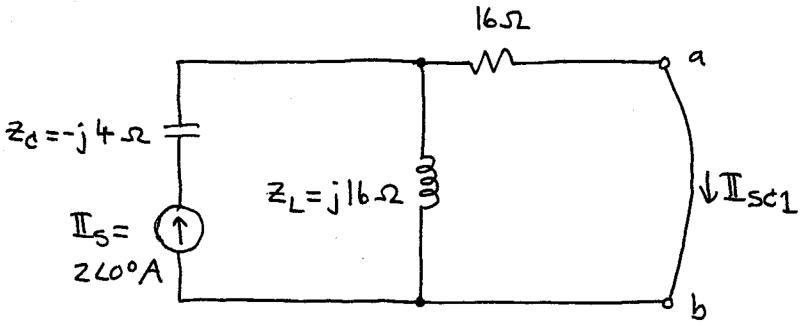


Since C is in series with current source, we have $V_x = -\mathbb{I}_S Z_C = j8V$ as before.

Thus, $2V_x = j16V$. We can now treat the dependent source as an independent source of $j16V$.

Now we use superposition to find $\mathbb{I}_{sc}$:

case I: $\mathbb{I}_S$ on, $2V_x$ off



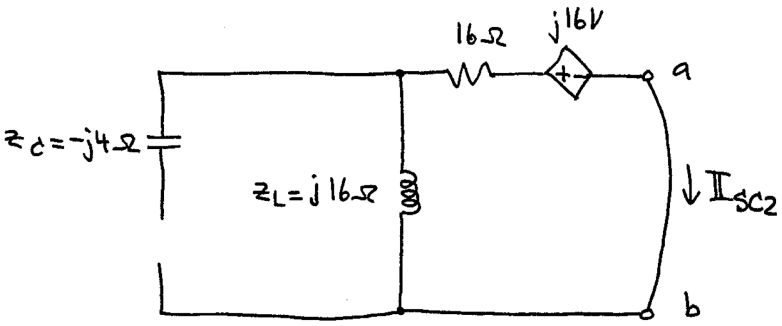
This is a current divider.

$$\mathbb{I}_{sc1} = \mathbb{I}_S \cdot \frac{Z_L}{Z_L + 16\Omega} = \frac{j16\Omega\, \mathbb{I}_S}{j16\Omega + 16\Omega} = \frac{j\,\mathbb{I}_S}{1+j}$$

$$= 2\angle 0°A \cdot \frac{j}{1+j}\,\frac{1-j}{1-j} = 2\angle 0°A\,\frac{1+j}{1^2+1^2} = 1+j\ A$$

$$\mathbb{I}_{sc1} = 1+j\ A$$

Case II: $\mathbb{I}_s$ off, $2V_x$ on



We have a V-loop on the right:

$$\mathbb{I}_{sc2} = \frac{-j16V}{16\Omega + z_L} = \frac{-j16V}{16\Omega + j16\Omega} = \frac{-j}{1+j}\ A$$

$$" = \frac{-j}{1+j}\frac{1-j}{1-j}\ A = \frac{-1-j}{1^2+1^2} = \frac{-1-j}{2}$$

Sum the results: $\mathbb{I}_{sc} = \mathbb{I}_{sc1} + \mathbb{I}_{sc2}$

$$= 1+j\ A + \frac{-1-j}{2}\ A$$

$$\mathbb{I}_{sc} = \frac{1+j}{2}\ A \text{ or } \frac{1}{\sqrt{2}}\angle 45^\circ A$$

$$z_{Th} = \frac{V_{Th}}{\mathbb{I}_{sc}} = \frac{j16V}{\frac{1+j}{2}} = \frac{16\angle 90^\circ V}{\frac{1}{\sqrt{2}}\angle 45^\circ A} = 16\sqrt{2}\angle 45^\circ\Omega$$

$$z_{Th} = 16 + j16\ \Omega$$