TOOL: Delaying a signal and the time it turns on by amount a causes its Laplace transform to be multiplied by an exponential (that corresponds to a phase shift proportional to frequency when $s = j\omega$):

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}\mathcal{L}\{f(t)\}$$

DERIV: Start with the definition of the Laplace transform of the left side:

$$\mathcal{L}\{f(t-a)u(t-a)\} = \int_{0^-}^{\infty} f(t-a)u(t-a)e^{-st}\,dt$$

Change variables to $\tau = t - a$:

$$\mathcal{L}\{f(t-a)u(t-a)\} = \int_{-a}^{\infty} f(\tau)u(\tau)e^{-s(\tau+a)}\,d\tau$$

Split the exponent into two pieces and observe that the $u(\tau)$ sets the integrand to zero until $\tau = 0$, allowing us to shift the lower limit to 0^-. (We will use 0^- because we want to pick up events that occur at exactly time a, such as a delta function, in the original integral. To be more precise, we could replace a by a^- everywhere, including the original integral and the statement of the identity. This, however, would depart from conventional notation.)

$$\mathcal{L}\{f(t-a)u(t-a)\} = \int_{0^-}^{\infty} f(\tau)u(\tau)e^{-s\tau}e^{-as}\,d\tau$$

We move the exponential involving a and s out front (since it is not a function of τ), and we observe that $u(\tau) = 1$ over the entire range of the integral and may be left out.

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}\int_{0^-}^{\infty} f(\tau)e^{-s\tau}\,d\tau$$

Finally, we change the variable of integration back to t to obtain the final identity statement.

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}\mathcal{L}\{f(t)\} \text{ ▪}$$