Ref: Nilsson & Riedel, Rev 6, Prob 12.16

Find $\mathcal{L}\{g(t)\}$ for a) $g(t) = -20e^{-5(t-2)}u(t-2)$

b) $g(t) = (8t-8)[u(t-1) - u(t-2)]$
$+ (24-8t)[u(t-2) - u(t-4)]$
$+ (8t-40)[u(t-4) - u(t-5)]$

ans: a) $\dfrac{-20e^{-2s}}{s+5}$ b) $\dfrac{8[e^{-s} - 2e^{-2s} + 2e^{-4s} - e^{-5s}]}{s^2}$

soln: a) Because we have $u(t-2)$, our signal is delayed. $\therefore$ We use the operational transform for delays:

$$\mathcal{L}\{f(t-a)u(t-a),\ a>0\} = e^{-as}F(s)$$

Here, we have $f(t) = -20e^{-5t}$ and $a = 2 > 0$.

$$g(t) = f(t-a)u(t-a) = -20e^{-5(t-2)}u(t-2)$$

Note that $f(t-a)$ results from substituting $t-a$ wherever t appears in $f(t)$.

Now, $F(s) = \mathcal{L}\{f(t)\} = \mathcal{L}\{-20e^{-5t}\} = -20\mathcal{L}\{e^{-5t}\}$

$$= -20 \cdot \frac{1}{s+5}$$

$$\therefore \mathcal{L}\{g(t)\} = G(s) = e^{-as}F(s) = \frac{-20e^{-2s}}{s+5}$$

b) We write this $g(t)$ as a sum of signals — one signal for each delay: $u(t-1)$, $u(t-2)$, $u(t-4)$, $u(t-5)$.

Then we write each signal so it looks like $f(t-a)u(t-a)$.

$$g(t) = (8t-8)u(t-1) + [(24-8t)-(8t-8)]u(t-2) + [(8t-40)-(24-8t)]u(t-4) - (8t-40)u(t-5)$$

$$= (8t-8)u(t-1) + (-16t+32)u(t-2) + (16t-64)u(t-4) - (8t-40)u(t-5)$$

Now write each line as $f(t-a)u(t-a)$:

$$g(t) = 8(t-1)u(t-1) - 16(t-2)u(t-2) + 16(t-4)u(t-4) - 8(t-5)u(t-5)$$

Now apply the operational transform for delays to each line.

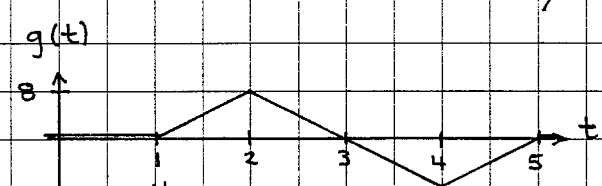
$$G(s) = 8e^{-s}F(s) - 16e^{-2s}F(s) + 16e^{-4s}F(s) - 8e^{-5s}F(s)$$

$F(s) = \mathcal{L}\{f(t) = t\,u(t)\} = \dfrac{1}{s^2}$

same $F(s)$ each time

or $G(s) = 8\left(e^{-s} - 2e^{-2s} + 2e^{-4s} - e^{-5s}\right)\dfrac{1}{s^2}$

Note: our original $g(t)$ is defined in piece-wise fashion. This lead to the use of delays: $u(t-a)$.



Note: We can write something like $t^2u(t-1)$ as $f(t-1)u(t-1)$:

$$t^2u(t-1) = [(t-1)^2 + 2(t-1) + 1]u(t-1)$$