EX: Find the Laplace transform of the following waveform:

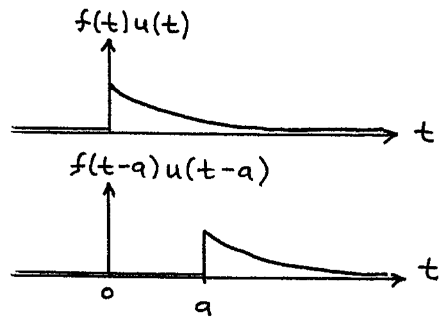
$$f(t) = e^{-3t}u(t-2)$$

sol'n: We apply the identity for signals that turn on after a delay:

$$\mathcal{L}\{f(t-a)\,u(t-a)\} = e^{-as}\,\mathcal{L}\{f(t)\,u(t)\}$$

(where $a > 0$)

pict:



Note that the signal has the same shape but is shifted to the right in the lower plot.

In order to apply the identity for delayed signals, we must find a way to write e^{-3t} as a function of $t-2$ to match the $u(t-2)$ delay.

Given a polynomial in t, the general approach for writing the polynomial in terms of $t-a$ is to replace the highest power of t with $t-a$ and adjust the coefficients of lower powers of t to compensate for the extra terms introduced into the polynomial.

ex: $p(t) = t^2 + 3t - 1$

Write $p(t)$ as a function of $t-a$ where $a = 4$.

sol'n: Replace t^2 with $(t-4)^2$ plus terms of lower order in t:

$$(t-4)^2 = t^2 - 8t + 16$$

Thus, $t^2 = (t-4)^2 + 8t - 16$.

$$\therefore p(t) = (t-4)^2 + 8t - 16 \quad + 3t - 1$$

Now replace the $8t + 3t = 11t$ with $11(t-4)$ plus a constant:

$$11(t-4) = 11t - 44$$

Thus, $11t = 11(t-4) + 44$

$$\therefore p(t) = (t-4)^2 + 11(t-4) + 44 - 1$$

or $p(t) = (t-4)^2 + 11(t-4) + 43$

Now we may write $p(t) = f(t-4)$ where

$$f(t) = t^2 + 11t + 43$$

Note: In this last step, we substitute t wherever $t-a$ appears in $p(t)$.

For the problem we are working on, we have e^{-3t}. We write $-3t$ as

$$-3t = -3(t-2) - 6$$

Thus $f(t) = e^{-3(t-2)-6}u(t-2)$

or $f(t) = e^{-6}e^{-3(t-2)}u(t-2)$

or $f(t) = g(t-2)$

where $g(t) = e^{-6}e^{-3t}u(t)$

Note: we use "$g(t)$" here since we have used "$f(t)$" already.

By the delay identity, we have

$$\begin{aligned}
\mathcal{L}\{e^{-3t}u(t-2)\} &= \mathcal{L}\{g(t-2)u(t-2)\} \\
&= e^{-2s}\mathcal{L}\{g(t)u(t)\} \\
&= e^{-2s}\mathcal{L}\{e^{-6}e^{-3t}u(t)\} \\
&= e^{-2s}e^{-6}\mathcal{L}\{e^{-3t}u(t)\} \\
&= e^{-2s}e^{-6}\mathcal{L}\{e^{-3t}\} \\
&= e^{-2s}e^{-6}\frac{1}{s+3}
\end{aligned}$$

or $\mathcal{L}\{e^{-3t}u(t-2)\} = e^{-2(s+3)}\dfrac{1}{s+3}$.