Apply initial and final value theorems to:

a) $F(s) = \dfrac{18s^2+66s+54}{(s+1)(s+2)(s+3)}$

b) $F(s) = \dfrac{25s^2+86s+40}{s(s+2)(s+4)}$

c) $F(s) = \dfrac{11s^2+172s+700}{(s+2)(s^2+12s+100)}$

d) $F(s) = \dfrac{56s^2+112s+5000}{s(s^2+14s+625)}$

ans: a) $f(0^+) = 18$ $\quad f(t\to\infty) = 0$

b) $f(0^+) = 25$ $\quad f(t\to\infty) = 5$

c) $f(0^+) = 11$ $\quad f(t\to\infty) = 0$

d) $f(0^+) = 56$ $\quad f(t\to\infty) = 8$

sol'n: a) Initial value theorem: $\lim\limits_{t\to 0^+} f(t) = \lim\limits_{s\to\infty} sF(s)$.

Highest power of s dominates in numerator or denominator as $s\to\infty$.

$$\therefore \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{\text{highest order } s \text{ term in numerator}}{\text{" " " " " denominator}}$$

Final value theorem: $\lim\limits_{t\to\infty} f(t) = \lim\limits_{s\to 0} sF(s)$

Write $sF(s)$ as $\dfrac{s^n}{s^m}\dfrac{p(s)}{q(s)} = s^{n-m}\dfrac{p(s)}{q(s)}$

where $p(s)$, $q(s)$ are polynomials with nonzero constant terms.

$$\text{Then } \lim_{s\to 0} sF(s) = \begin{cases} 0 & n > m \\ \infty & n < m \\ \dfrac{p(0)}{q(0)} & n = m \end{cases}$$

Note: $p(0)$ = constant term in $p(s)$
$q(0)$ = " " " $q(s)$

For (a) we have $sF(s) = \dfrac{s\,(18s^2 + 66s + 54)}{(s+1)(s+2)(s+3)}$

$f(0^+) = \lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \dfrac{18s^3}{s^3}$ (highest order s terms from numerator & denominator)

or $f(0^+) = 18$

$f(t\to\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \dfrac{\overset{s}{\overset{\shortparallel}{s^1}}}{\underset{\underset{1}{\shortparallel}}{s^0}} \dfrac{18s^2+66s+54}{(s+1)(s+2)(s+3)}$

or $f(t\to\infty) = 0$ (since $n < m$)

b) $sF(s) = \dfrac{25s^2 + 86s + 40}{(s+2)(s+4)}$

$f(0^+) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \dfrac{25s^2}{s^2} = 25$

$f(t\to\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \dfrac{\overset{1}{\overset{\shortparallel}{s^0}}}{\underset{\underset{1}{\shortparallel}}{s^0}} \dfrac{25s^2+86s+40}{(s+2)(s+4)} = \dfrac{40}{2\cdot 4} = 5$

c) $sF(s) = \dfrac{s\,(11s^2 + 172s + 700)}{(s+2)(s^2 + 12s + 100)}$

$f(0^+) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \dfrac{11s^3}{s^3} = 11$

$f(t\to\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \dfrac{s^1}{s^0} \dfrac{11s^2+172s+700}{(s+2)(s^2+12s+100)} = 0$ since $n > m$

d) $sF(s) = \dfrac{56s^2 + 112s + 5000}{s^2 + 14s + 625}$

$f(0^+) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \dfrac{56s^2}{s^2} = 56$

$f(t\to\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \dfrac{s^0}{s^0} \dfrac{56s^2+112s+5000}{s^2+14s+625} = \dfrac{5000}{625} = 8$

Note: It is possible to get $f(0^+) = \infty$ and/or $f(t\to\infty) = \infty$.

Note: For initial value theorem, we also should verify that all poles are in left-half plane. Otherwise, $f(t\to\infty) = \infty$. Unstable system.