EX: Derive the following Laplace transform pair:

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

SOL'N: Use induction and the following identity:

$$\mathcal{L}\{tf(t)\} = -\frac{d}{ds}\mathcal{L}\{f(t)\}$$

From a basic table of Laplace transform pairs, we verify that the transform pair is valid for $n = 0$ and $n = 1$:

$$\mathcal{L}\{t^0\} = \mathcal{L}\{1\} = \mathcal{L}\{u(t)\} = \frac{1}{s} = \frac{0!}{s^{0+1}}$$

$$\mathcal{L}\{t^1\} = \frac{1}{s^2} = \frac{1!}{s^{1+1}}$$

Now assume the transform pair is true for n (and then show that it holds for $n + 1$).

$$\text{Assume } \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

Then our identity says

$$\mathcal{L}\{t \cdot t^n\} = -\frac{d}{ds}\frac{n!}{s^{n+1}}$$

Calculating the derivative yields the following:

$$\mathcal{L}\{t \cdot t^n\} = -n!\frac{-(n+1)}{s^{n+2}}$$

This simplifies to the transform pair formula for $n + 1$:

$$\mathcal{L}\{t^{n+1}\} = \frac{(n+1)!}{s^{n+2}}$$

It follows, by the principle of mathematical induction, that the transform pair must be valid for all nonnegative n.