Linear Algebra —

20 Apr 1989 Graham - Schmidt Orthonormalization

Neil E Cotter

Oftentimes, one wishes to create a set of vectors that are all orthogonal to a given vector. This can be done by picking a set of vectors at random and subtracting out projections to leave only perpendicular vectors. The resulting vectors are then normalized by dividing them by their length.

Note that for N-dimensional vectors, at most N different vectors can be orthogonal to each other. Incidentally, the N vectors form a basis for the space in that any other vector can be written as a linear combination of these vectors.

Simplest example is 2-dimensional case ($N=2$):

Assume $\vec{y}$ is given.
Pick an $\vec{x}$ at random.

Subtract the projection $P_y \vec{x}$ from $\vec{x}$ to obtain a vector $\perp$ to y:

$$\vec{x} - P_y \vec{x} = \vec{x} - \left(\vec{x} \circ \frac{\vec{y}}{|\vec{y}|} \right) \frac{\vec{y}}{|\vec{y}|} = \perp_y \vec{x}$$ *see p.8

Now normalize this vector (i.e. make its length be <u>one</u>):

$\vec{z} \equiv \dfrac{\perp_y \vec{x}}{|\perp_y \vec{x}|}$ is our desired vector: $\vec{z} \perp \vec{y}$ and $|\vec{z}| =$

Note that for any vector $\vec{x}$, $\dfrac{\vec{x}}{|\vec{x}|}$ has unit length.

If desired, the normalization step may be left out.