Ex: Consider a brush DC motor described by the following equations:

$$0 = v - Ri - K\omega$$

$$J\frac{d\omega}{dt} = Ki - C$$

$$\frac{d\theta}{dt} = \omega$$

The parameters for the motor are as follows:

L = 0 H (the motor has no inductance)

$R = 2\ \Omega$

$K = 4$ Nm/A

$J = 0.8$ kg m^2

$C = 8$ Nm

The voltage applied to the motor is constant at 12 V for a very long time before time $t = 0$. Thus, the motor is running at constant velocity just before $t = 0$.

At $t = 0$, the voltage applied to the motor drops instantly to 0 V and stays at 0 V from then on.

a) Calculate the value of $\omega(t = 0^-)$. Note that this is just before time $t = 0$.

b) Calculate the value of the power, $p_J(t = 0^+)$, arising from the moment of inertia of the motor and its changing velocity immediately after time $t = 0$. Note that this is just after time $t = 0$.

sol'n: a) At $t=0^-$, the motor is in steady-state.

Thus, $\frac{d\omega}{dt} = 0$, (constant velocity).

$\therefore\ 0 = Ki - C$ and $i = \frac{C}{K}$

Substituting into 1st eq'n, we have

$0 = v - Ri - K\omega$ where $i = \frac{C}{K}$

After some algebra, $\omega = \frac{V - RC/K}{K}$,

or $\omega = \frac{12V - 2\Omega \cdot 8Nm / 4\ Nm/A}{4\ Nm/A \text{ or } 4Vs}$

$\omega = 2\ r/s$

b) Power from changing velocity of motor is

$$P_J(t=0^+) = J\omega\Big|_{t=0^+} \cdot d\omega/dt\Big|_{t=0^+}$$

We know the speed of the motor will not change instantly. Thus, $\omega(t=0^+) = \omega(t=0^-)$, and $\omega(t=0^-) = 2\ r/s$ from (a).

From the 2nd motor eq'n, $J\frac{d\omega}{dt}\Big|_{t=0^+} = Ki\Big|_{t=0^+} - C$.

From the 1st motor eq'n, $0 = v\Big|_{t=0^+} - Ri\Big|_{t=0^+} - K\omega\Big|_{t=0^+}$

$v(0^+) = 0\,V$ from problem statement.

$\omega(0^+) = 2\ r/s$ as noted earlier.

$$\therefore\ i(t=0^+) = \frac{0V - K\cdot 2r/s}{R} = -\frac{4\ Nm/A\cdot 2r/s}{2\Omega}$$

$$i(t=0^+) = -4\ A$$

$$P_J(0^+) = J\cdot\omega(0^+)\cdot\left.\frac{d\omega}{dt}\right|_{t=0^+} = \underbrace{(K\,i(0^+) - C)}_{\left.J\cdot\frac{d\omega}{dt}\right|_{t=0^+}}\omega(0^+)$$

$$P_J(0^+) = [4\ Vs\cdot(-4A) - 8\ Ws]\,2\,r/s$$

$$P_J(0^+) = -24\cdot 2\ W = -48\ W$$

Note: the motor acts like a generator producing $i = -4A$ at $t = 0^+$.