

Tool: DC Motor Models

The DC motor eq'ns are

$$\frac{di}{dt} = \frac{1}{L} v - \frac{R}{L} i - \frac{K}{L} \omega \quad (1)$$

$$\frac{d\omega}{dt} = \frac{K}{J} i - \frac{1}{J} \tau_{LF} \quad (2)$$

We consider $\tau_{LF} = B\omega$ unless otherwise noted.

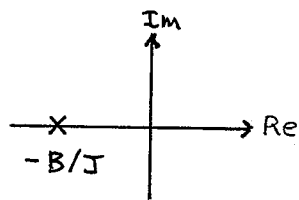
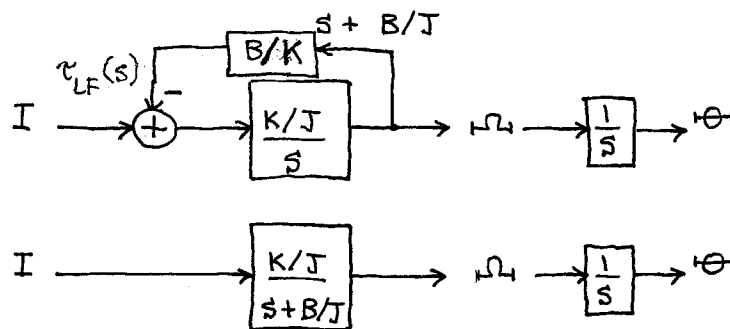
Current Control

Current control means we use only eq'n (2).

$$s\Omega(s) = \frac{K}{J} I(s) - \frac{1}{J} B\Omega(s)$$

$$\left(s + \frac{B}{J}\right)\Omega(s) = \frac{K}{J} I(s)$$

$$\Omega(s) = \frac{K/J}{s + B/J} I(s)$$



Voltage Control

For voltage control, we use both eqns (1) and (2).

Solving (1) for $I(s)$ in Laplace domain and substituting into (2) yields

$$\left(s^2 + \frac{R}{L}s + \frac{K^2}{JL} \right) \Omega(s) = \frac{K}{JL} V(s) - \frac{(sL+R)}{JL} B \Omega(s)$$

Note: substitute $\tau_{LF}(s)$ for $B \Omega(s)$ if desired.

Often we have a much faster time constant $\tau_e \equiv L/R$ for charging L than the time constant $\tau_m \equiv \frac{K^2}{RJ} + \frac{B}{J}$.

In that case, we have (for the denominator of $\frac{\Omega(s)}{V(s)}$)

$$s^2 + \left(\frac{R}{L} + \frac{B}{J} \right) s + \frac{K^2 + RB}{JL} \approx \left(s + \frac{R}{L} \right) \left(s + \frac{1}{J} \left[\frac{K^2}{R} + B \right] \right)$$

$$\therefore \Omega(s) \approx \frac{K/JL}{\left(s + \frac{R}{L} \right) \left(s + \frac{1}{J} \left[\frac{K^2}{R} + B \right] \right)} V(s)$$

