Ex: Consider a brush DC motor described by the following equations:

$$0 = v - Ri - K\omega$$

$$J\frac{d\omega}{dt} = Ki - C$$

$$\frac{d\theta}{dt} = \omega$$

The parameters for the motor are as follows:

L = 0 H (the motor has no inductance)

$R = 2\ \Omega$

$K = 4$ Nm/A

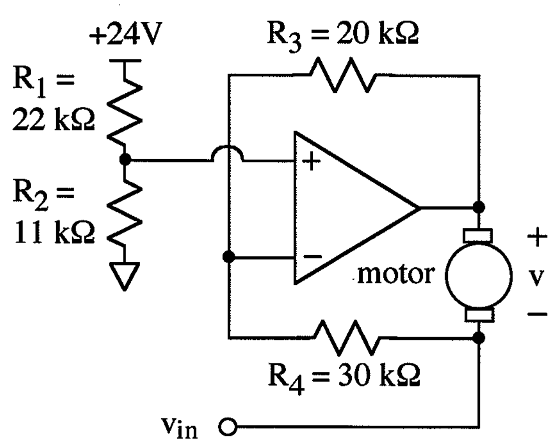
$J = 0.8$ kg m^2

$C = 8$ Nm

The voltage applied to the motor is constant at 12 V for a very long time before time $t = 0$. Thus, the motor is running at constant velocity just before $t = 0$.

At $t = 0$, the voltage applied to the motor drops instantly to 0 V and stays at 0 V from then on.

The motor control circuit includes the following op-amp circuit:



Assuming the op-amp is ideal and responds instantly, find the value of $v_{in}(t = 0^+)$. Note that this is just after time $t = 0$. Show your work.

sol'n For the ideal op-amp, we have $V_+ = V_-$.

We have a V-divider that sets V_+ at

$$V_+ = 24V \cdot \frac{R_2}{R_1+R_2} = 24V \cdot \frac{11k\Omega}{11k\Omega+22k\Omega} = 8V.$$

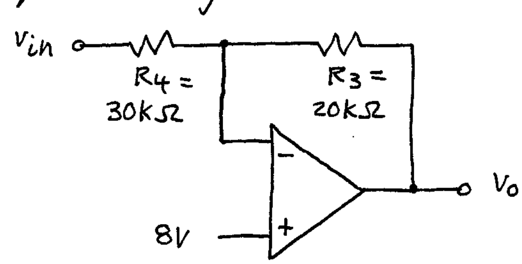
Note: No current flows into op-amp inputs.

Given v_{in}, we have the motor acting as a load on the op-amp (which does not affect the op-amp output) with its other side connected to v_{in} which we treat as a V-src.

Since R_4 is connected to v_{in} (treated as a V-src), the current thru R_4 is unaffected by the motor.

Thus, we conclude that we may ignore the motor. That is, we may pretend it is not there and calculate the voltage drop from the output of the op-amp to v_{in}.

Redrawing the circuit, we have a much simpler diagram:



$V = V_o - v_{in}$

Using superposition and turning on v_{in} and the +8V src's one at a time allows us to apply the standard positive and negative gain formulas:

$$v_o = v_{o1} + v_{o2} \quad \text{where}$$

$$v_{o1} = v_{in}\left(-\frac{R_3}{R_4}\right) = v_{in}\left(-\frac{2}{3}\right)$$

$$v_{o2} = 8V\left(1+\frac{R_3}{R_4}\right) = 8V\cdot\left(1+\frac{2}{3}\right) = 8V\cdot\frac{5}{3}$$

$$v_o = -\frac{2}{3}v_{in} + \frac{40}{3}V$$

Thus, $v = v_o - v_{in} = -\frac{5}{3}v_{in} + \frac{40}{3}V = 0$ at $t=0^+$.
since $v(0^+)=0$

$\therefore v_{in} = 8V$