EX: Consider the steady-state torque equation for an induction motor, [1], with the parameters listed below:

$$\tau_e = \frac{n_p M^2}{R_R} I_S^2 \frac{S}{1 + (T_R S)^2}$$

where
$\tau_e \equiv$ torque (Nm)
$n_p \equiv$ number of pole pairs
$M \equiv$ mutual inductance of rotor and stator coils (H)
$R_R \equiv$ rotor resistance (Ω)
$I_S \equiv$ magnitude of stator current (A)
$S \equiv$ slip (rad/s)
$T_R \equiv$ rotor time constant = L_R/R_R (s)
$L_R \equiv$ rotor inductance (H)

Use Matlab to make a plot of slip frequency, S, versus steady-state electrical frequency, ω_e. Superimpose a plot of ω versus ω_e. Assume the steady-state torque equals friction $B\omega$. (Rewrite ω as $\omega_e - S$ when solving for S. After you find S, use this equation again to calculate ω.) Use values of ω_e in the range [1, 30] rad/s.

NOTE: The equation to be solved is a cubic in S. Use the smallest real root as the value of S. The Matlab function `roots([1,a,b,c])` finds the roots of the following cubic polynomial:

$$S^3 + aS^2 + bS + c$$

Motor parameters:

$$L_R = \frac{2}{100}\text{H} \qquad R_R = \frac{2}{10}\Omega \qquad M = \frac{1}{100}\text{H}$$

$$I_S = 200\text{A} \qquad B = 20\text{Nms} \qquad n_p = 1$$

REF: [1] Marc Bodson, "Control of Electric Motors," 2004, University of Utah ECE Dept., eqn 4.62 p. 130.

SOL'N: Define a convenient constant to simplify notation

$$K \equiv \frac{n_p M^2}{R_R} I_S^2 = 20 \text{ Nms}$$

Setting $\tau_e = B\omega$ and substituting for ω in terms of S, we have

$$B(\omega_e - S) = K \frac{S}{1 + (T_R S)^2}$$

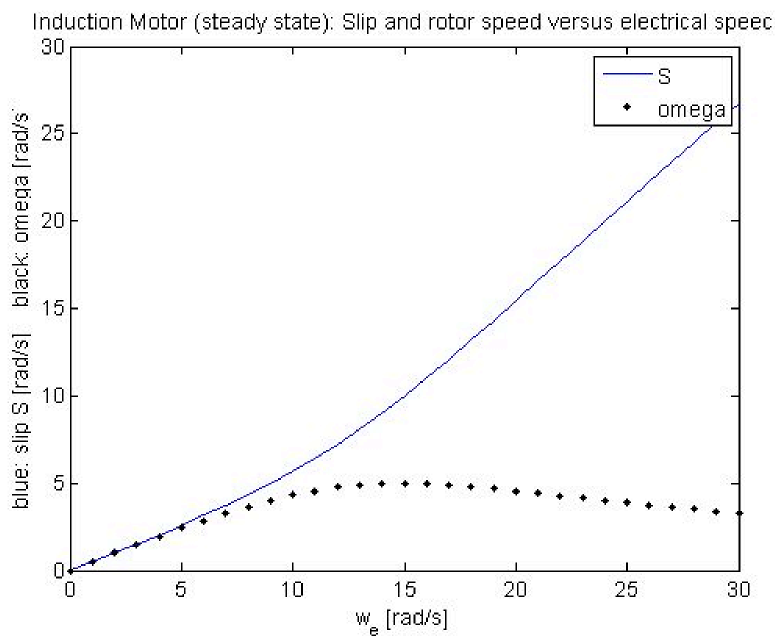
Rearranging gives

$$B(\omega_e - S)[1 + (T_R S)^2] = KS$$

Simplifying, we have a cubic equation for S:

$$S^3 - \omega_e S^2 + \frac{2}{T_R^2} S - \frac{\omega_e}{T_R^2} = 0$$

We use Matlab to calculate S versus ω_e and ω versus ω_e, (see code below).



We observe that there is a peak motor speed for $\omega_e \approx 15$ rad/s.

```
% ECE5570F05prob3soln.m
%
% Matlab code for plotting slip frequency S versus electrical freq w_e.
% For induction motor.  Uses eqn 4.62 from "Control of Electric Motors"
% by Marc Bodson.

% Set values of motor parameters.
L_R = 2/100;     % (H)     = inductance of rotor
R_R = 2/10;      % (ohms) = resistance of rotor
M = 1/100;       % (H)     = mutual inductance of stator and rotor
I_s = 200;       % (A)     = current in stator
J_motor = 5/100; % (kgm^2) = moment of inertia of motor (not used)
np = 1;          % (-)      = number of pole pairs

% Calculate rotor time constant.
T = L_R / R_R;

% Use figure 1 for all plots
figure(1)

% Create empty w_e_vec array so Matlab accepts later statements.
w_e_vec = [];

% Create empty Svec array so Matlab accepts later statements.
Svec = [];

% Plot S for values of w_e in range [1,5] rad/s.
for w_e = 0:1:30

  % Build w_e vec.
  w_e_vec = [w_e_vec, w_e];

  % Set coeffs of polynomial to be solved.  Polynomial eqn is:
  %  S^3 - w_e * S^2 + 2/T^2 * S - w_e/T^2 = 0
  poly_coeffs = [1, -w_e, 2/T^2, -w_e/T^2];

  % Find roots of polynomial.
  S_vals = roots(poly_coeffs);

  % Find real roots.
  real_S_vals = S_vals(find(abs(imag(S_vals)) < eps));

  % Use smallest real root.
  S_min = min(real_S_vals);

  % Add answer to array of S values vs w_e.
  Svec = [Svec, S_min];

end

% Calculate motor speed, omega.
omega = w_e_vec - Svec;

  % plot results
  plot(w_e_vec, Svec, 'b-', w_e_vec, omega, 'k.')
  xlabel('w_e [rad/s]')
  ylabel('blue: slip S [rad/s]    black: omega [rad/s]')
```