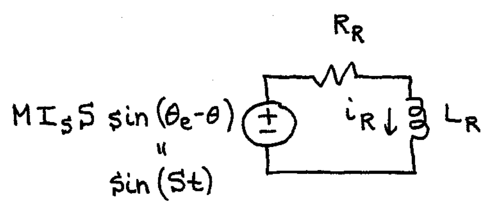
Induction Motor:

$$L_R \frac{di_R}{dt} = \overset{0}{\overset{\shortparallel}{v_R}} - R_R i_R + M I_S \sin(\theta_e - \theta) \underset{\text{slip } S}{\underset{\equiv}{(\omega_e - \omega)}}$$

$$\tau_e = M I_S i_R \sin(\theta_e - \theta)$$

Q. If $S$ becomes too high, does the magnetic field get so far ahead of the rotor that it pulls it backward?

A. Consider constant $S$ and $\omega_e$. Suppose $\omega = 0$. In this case, we have an electrical equivalent circuit with R and L driven by a sinusoidal voltage source.




The response of the circuit will be sinusoidal and at the same frequency as the source. Using phasors:

$$\mathbb{I}_R = \frac{\overbrace{-j M \mathbb{I}_S S}^{\text{phasor for } V \text{ src}}}{R_R + j\omega L}$$

where $\omega = S$

Writing $\mathbb{I}_R$ in terms of magnitude and phase:

$$\mathbb{I}_R = \frac{M \mathbb{I}_S S}{\sqrt{R_R^2 + (SL)^2}} \angle \overbrace{-90^\circ}^{\text{from } -j} - \underbrace{\tan^{-1}\left(\frac{SL}{R}\right)}_{\text{between } 0^\circ \text{ and } +90^\circ}$$

Thus, $i_R(t) = \dfrac{MI_s S}{\sqrt{R_R^2 + (SL)^2}} \cos(St + \varphi)$

where $\varphi = -90° - \tan^{-1}\left(\dfrac{SL}{R}\right)$, $-180° \le \varphi \le -90°$

Our torque is

$$\tau_e = MI_s i_R \sin(\theta_e - \theta)$$

$$= MI_s \frac{MI_s S}{\sqrt{R_R^2 + (SL)^2}} \cos(St + \varphi) \cos(St - 90°)$$

$$= \frac{(MI_s)^2}{\sqrt{R_R^2 + (SL)^2}} S\left[\frac{1}{2}\cos(2St + \varphi - 90°) + \frac{1}{2}\cos(\varphi + 90°)\right]$$

If we have X and Y coils on rotor, the $\cos(2St + \varphi - 90°)$ will be canceled out, but the $\frac{1}{2}\cos(\varphi + 90°)$ will be doubled.

Thus, we will have

$$\tau_e = \frac{(MI_s)^2}{\sqrt{R_R^2 + (SL)^2}} S \cos(\varphi + 90°)$$

But $\varphi + 90° = -\tan^{-1}\left(\dfrac{SL}{R}\right)$

$$-90° \le \varphi \le 0°$$

This means the torque is always positive since $\cos\varphi \ge 0$ for $-90° \le \varphi \le 0°$.
Also, $i_R$ lags the rotation of the magnetic field by at most 90°. $\therefore \tau_e > 0$ always

Steady State:

$$\mathbb{I}_R = \frac{-jMS}{R_R + jSL_R}\mathbb{I}_S$$

and $$\mathbb{V}_S = (R_S + j\omega_e L_S)\mathbb{I}_S + j\omega_e M \mathbb{I}_R$$

Substituting for $\mathbb{I}_R$:

$$\mathbb{V}_S = (R_S + j\omega_e L_S)\mathbb{I}_S + \frac{\omega_e S M^2}{R_R + jSL_R}\mathbb{I}_S$$

or $$\frac{\mathbb{V}_S}{\mathbb{I}_S} = R_S + j\omega_e L_S + \underbrace{\frac{\omega_e S M^2}{R_R + jSL_R}}_{\text{reflected } z \text{ from rotor}} \equiv z_S$$

Define $\tau_R \equiv \frac{L_R}{R_R}$ $\tau_S \equiv \frac{L_S}{R_S}$

$$\sigma \equiv 1 - \frac{M^2}{L_S L_R}$$

Then $$z_S = \frac{(R_S - \omega_e \sigma L_S \tau_R S) + j(\omega_e L_S + R_S \tau_R S)}{1 + j\tau_R S}$$

$$z_S \approx j\omega_e L_S \frac{1 + j\sigma\tau_R S}{1 + j\tau_R S} \quad \text{for small } R_S$$

For $\mathbb{V}_S$ constant, $\mathbb{I}_S = \frac{\mathbb{V}_S}{z_S}$ behaves like $-j\,\frac{1+j\tau_R S}{1+j\sigma\tau_R S}\cdot$ constant