Synchronous Motor Eq'ns:

$$\frac{di_A}{dt} = \frac{1}{L}v_A - \frac{R}{L}i_A + \frac{K}{L}\omega\sin(n_p\theta)$$

$$\frac{di_B}{dt} = \frac{1}{L}v_B - \frac{R}{L}i_B - \frac{K}{L}\omega\cos(n_p\theta)$$

$$\frac{d\omega}{dt} = -\frac{K}{J}i_A\sin(n_p\theta) + \frac{K}{J}i_B\cos(n_p\theta) - \tau_{LF}$$

Synchronous Motor Eq'ns after DQ transformation:

$$\frac{di_d}{dt} = \frac{1}{L}v_d - \frac{R}{L}i_d + n_p\omega i_q$$

$$\frac{di_q}{dt} = \frac{1}{L}v_q - \frac{R}{L}i_q - n_p\omega i_d - \frac{K}{L}\omega$$

$$\frac{d\omega}{dt} = \frac{K}{J}i_q - \tau_{LF}$$

DQ transformation: (similar eq'ns for voltages)

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos(n_p\theta) & \sin(n_p\theta) \\ -\sin(n_p\theta) & \cos(n_p\theta) \end{bmatrix}\begin{bmatrix} i_A \\ i_B \end{bmatrix} = U(\underset{\text{or } n_p\theta}{\theta})\begin{bmatrix} i_A \\ i_B \end{bmatrix}$$

DQ^{-1} transformation:

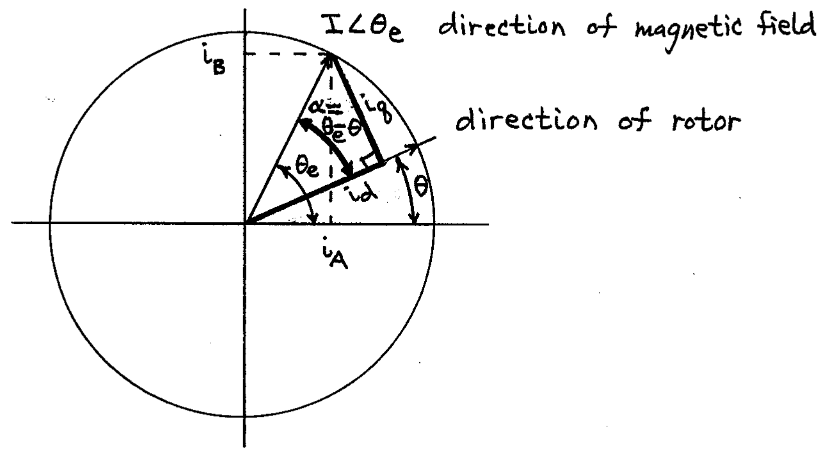
$$\begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} \cos(n_p\theta) & -\sin(n_p\theta) \\ \sin(n_p\theta) & \cos(n_p\theta) \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} = U^{-1}(\underset{\text{or } n_p\theta}{\theta})\begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

Note: $U^{-1}(\theta) = U(-\theta) = U^T(\theta)$

$\theta \equiv$ angle of rotor

$n_p \equiv$ # of pole pairs

DQ transformation in terms of angles (currents)



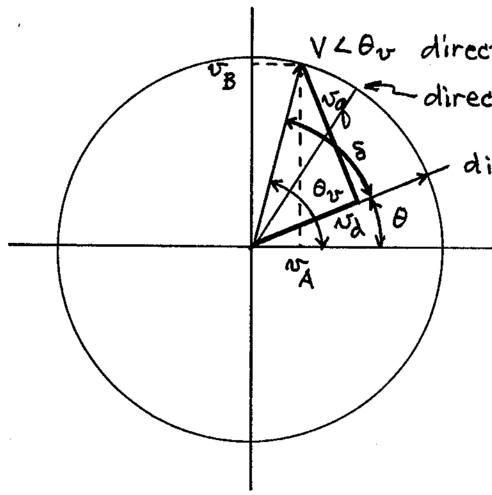
$$i_A = I\cos(\theta_e)$$
$$i_B = I\sin(\theta_e)$$

$$i_d = I\cos(\theta_e - \theta) = I\cos(\alpha)$$
$$i_q = I\sin(\theta_e - \theta) = I\sin(\alpha)$$

Note: replace θ with $n_p\theta$ for case of n_p pole pairs.

From diagram, $\sqrt{i_A^2 + i_B^2} = \sqrt{i_d^2 + i_q^2} = I^2$

DQ transformation in term of angles (voltages)



$$v_A = V\cos(\theta_v)$$
$$v_B = V\sin(\theta_v)$$

$$v_d = V\cos(\theta_v - \theta) = V\cos(\delta)$$
$$v_q = V\sin(\theta_v - \theta) = V\sin(\delta)$$

Note: replace θ with $n_p\theta$ for case of n_p pole pairs

From diagram, $\sqrt{v_A^2 + v_B^2} = \sqrt{v_d^2 + v_q^2} = V^2$

Note: $\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \underset{\text{or } n_p\theta}{\mathcal{U}(\theta)} \begin{bmatrix} v_A \\ v_B \end{bmatrix}$

Curiously, the DQ transformation is only a function of θ (or $n_p\theta$) but not δ.