28 Mar 1990 Perceptron — Circular Classification Regions

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modified

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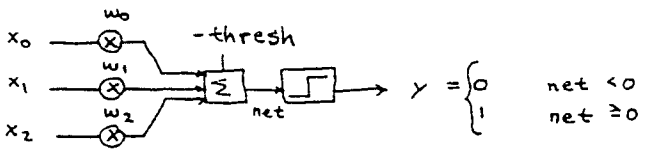
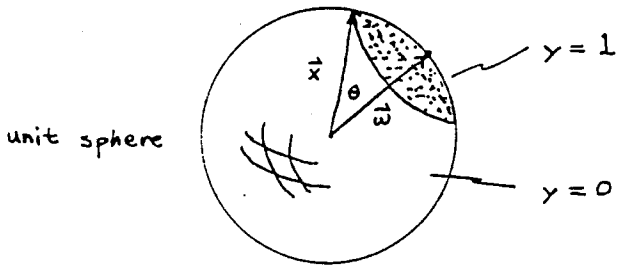


Suppose $|\vec{x}| = 1$, $|\vec{w}| = 1$. Requires unit length

Then response region for neuron is circular region on sphere; centered on $\vec{w}$:



Note that $|\vec{x}| = |\vec{w}| = 1$ implies $\vec{x}, \vec{w}$ lie on unit

i.e. neuron measures distance from $\vec{w}$ to $\vec{x}$.

$$\text{dist}^2 = |\vec{w} - \vec{x}|^2 = (\vec{w} - \vec{x}) \circ (\vec{w} - \vec{x})$$

$$= \vec{w} \circ \vec{w} - 2\vec{x} \circ \vec{w} + \vec{x} \circ \vec{x}$$

$$= |\vec{w}|^2 + |\vec{x}|^2 - 2\vec{x} \circ \vec{w}$$

$$= 1 + 1 - 2\vec{x} \circ \vec{w}$$ ← computed by neuron

$$= 2 - 2\vec{x} \circ \vec{w}$$

$$= 2 - 2|\vec{x}||\vec{w}| \cos\theta$$ 2nd time we used $|\vec{x}| = |\vec{w}| = 1$

$$= 2 - 2\cos\theta$$

Boundary is where $\vec{x} \circ \vec{w} = \text{thresh}$

$= \cos\theta$

$\therefore$ $\vec{x} \circ \vec{w} > \text{thresh}$ (i.e. $y = 1$ output) when

$$\theta \leq \cos^{-1}(\text{thresh})$$