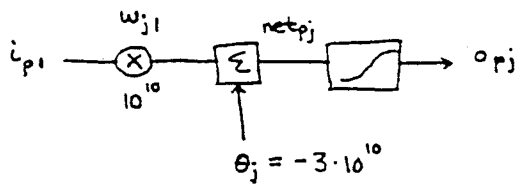
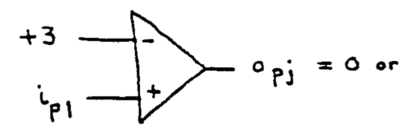
11 May 1989
Neil E Cotter

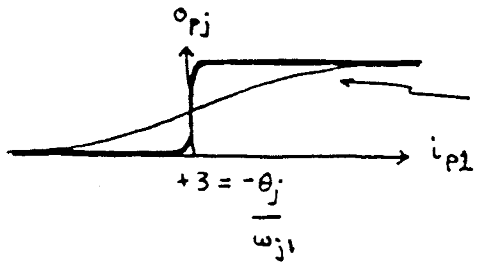
Perceptrons — Logic Gates
What One Neuron Can Compute

ex: Comparator





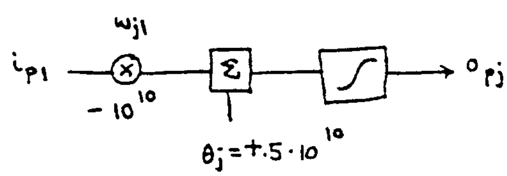
equivalent circuit

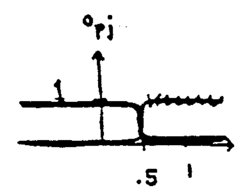


- Large synaptic weight is equivalent to using steeper sigmoid (bold line), causing neuron output to be almost binary 0 or 1.
- Transition point is at $\frac{-\theta_j}{w_{j1}}$

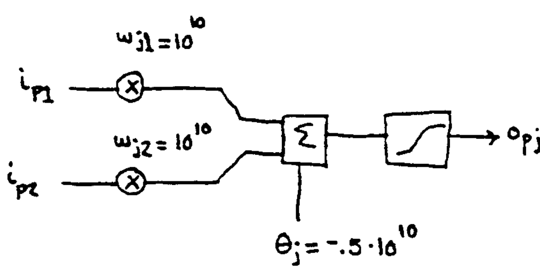
ex: Logic Gates

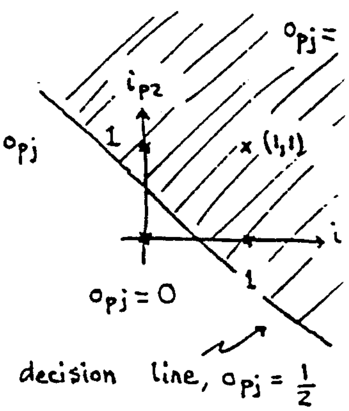
Inverter:





OR gate:





- These gates accept analog inputs. So do gates found in electronic circuits.
- Define the "decision line" to be the set of <u>input</u> points that give $o_{pj} = \frac{1}{2}$.

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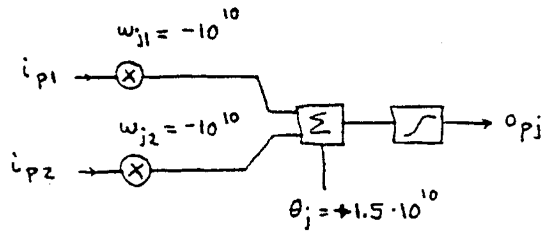
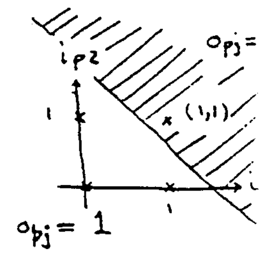
Perceptrons – Logic Gates (cont.)

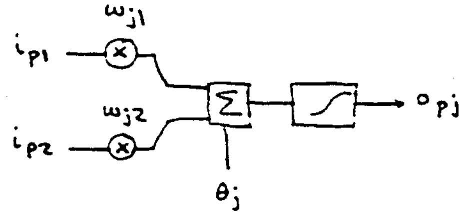
NAND gate:

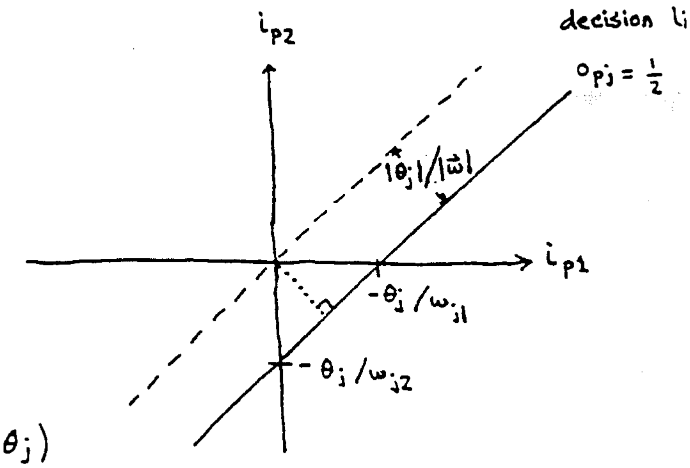




- Since we can make logic gates out of neurons, with enough neurons we can make a computer.

General Case of Two-input Neuron





Geometric Analysis:

$$o_{pj} = f(w_{j1} i_{p1} + w_{j2} i_{p2} + \theta_j)$$

$$o_{pj} = \frac{1}{2} \quad \text{when} \quad w_{j1} i_{p1} + w_{j2} i_{p2} + \theta_j = 0$$

$$\text{or} \quad \vec{w} \circ \vec{i} + \theta_j = 0 \qquad \vec{w} = \begin{bmatrix} w_{j1} \\ w_{j2} \end{bmatrix} \quad \vec{i} = \begin{bmatrix} i_{p1} \\ i_{p2} \end{bmatrix}$$

claim: Iff $\vec{i}$ lies on the decision line then the projection (length), $|P_{\vec{w}}\,\vec{i}|$, of $\vec{i}$ onto $\vec{w}$ has length $\frac{|\theta_j|}{|\vec{w}|}$.

proof:

$$\left|P_{\vec{w}}\,\vec{i}\right| = \frac{|\vec{w} \circ \vec{i}|}{|\vec{w}|}$$

$$\vec{w} \circ \vec{i} + \theta_j = 0 \quad \Rightarrow \quad \frac{|\vec{w} \circ \vec{i}|}{|\vec{w}|} = \frac{|\theta_j|}{|\vec{w}|} .$$

It follows from this claim that $\vec{w}$ is perpendicular to decision