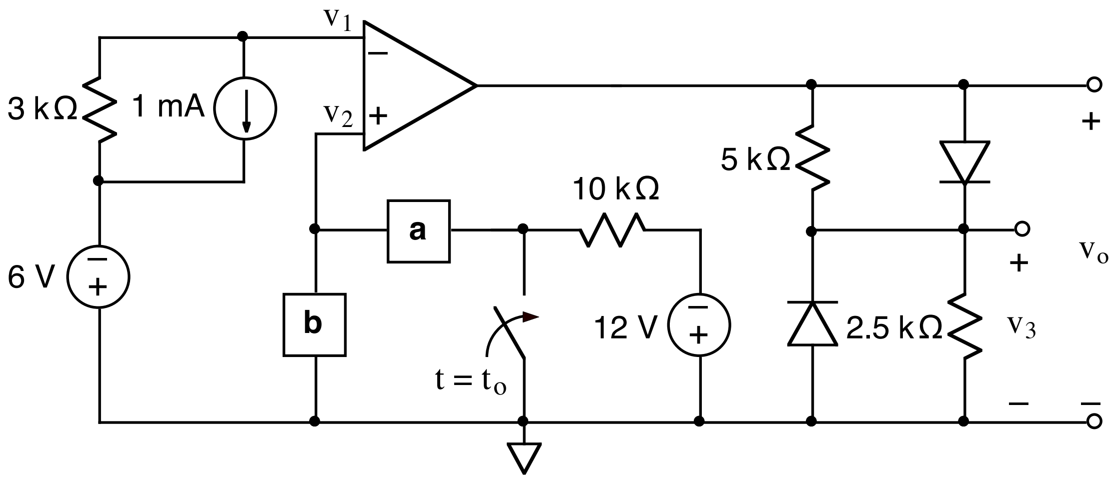
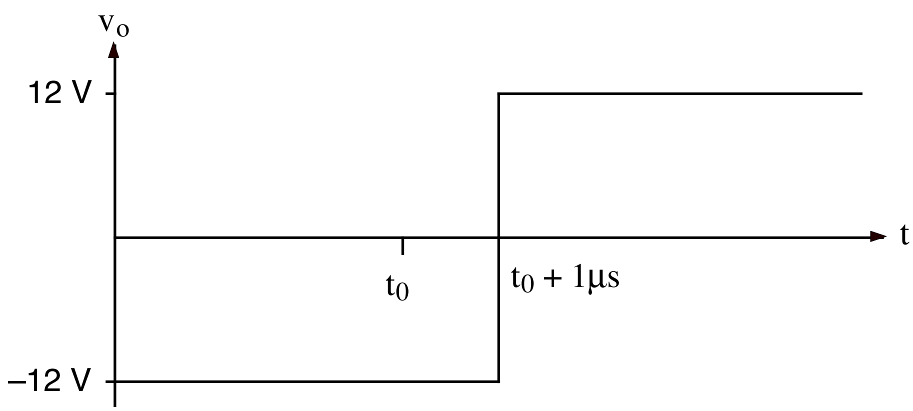
Ex:



Rail voltage = ±12 V

After being open for a long time, the switch closes at $t = t_o$.



a) Choose either an R or L to go in box **a** and either an R or L to go in box **b** to produce the $v_o(t)$ shown above. (Note that v_o stays high forever after $t_o + 1$ µs.) Specify which element goes in each box and its value.

b) Sketch $v_1(t)$, showing numerical values appropriately.

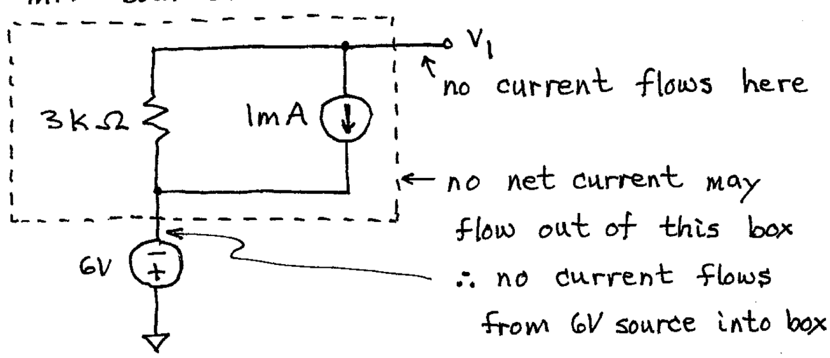
c) Sketch $v_2(t)$, showing numerical values appropriately.

d) Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 1$ µs, and for $t_o + 1$ µs $< t$. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

sol'n: a) v_o starts out with a negative value, implying $v_2 < v_1$ for the op-amp inputs.

If we place an L in box **b**, then at time t_o^-, (when the L acts like a wire), we would have $v_2 = 0V$.

Thus, we need to know the value of v_1. Since the inputs of the op-amp act like sensors that draw no current, we can solve for v_1 by considering only the 6V source, the 3 kΩ resistor, and the 1 mA source.

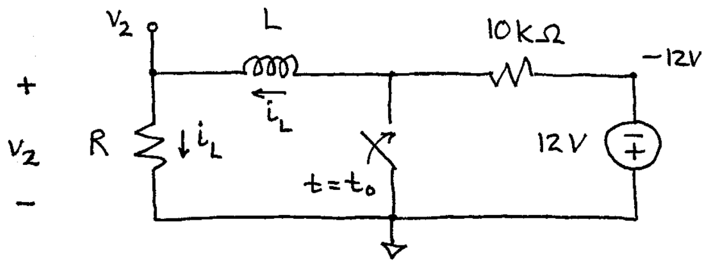


We have $v_1 = -6V - 1mA \cdot 3k\Omega = -9V$. Note that the 1 mA must flow through the 3kΩ resistor since no current flows into the op-amp.

We now see that an L in box **b** is impossible, as it would yield $v_2 = 0V > v_1$, implying $v_o > 0V$ at time $t = t_o^-$.

Thus, we must have an R in box **b**. To have a dynamic time response, we must have an L in **a**.

The circuit that determines v_2 is as follows:



We may assume $t_0 = 0$ for convenience.

At $t = 0^-$, the circuit has reached equilibrium and L = wire.

$$i_L(0^-) = \frac{-12V}{R + 10k\Omega}$$

$$v_2(0^-) = i_L(0^-) \cdot R = -12V \cdot \frac{R}{R + 10k\Omega}$$

We need to have $v_2(0^-) < -9V$ at $t = 0^-$. This means that we must use $R > 30k\Omega$ since $-9V$ is 3/4 of $-12V$ and $R = 30k\Omega$ gives $\frac{R}{R + 10k\Omega} = \frac{3}{4}$. We return to this later.

Since i_L can't change instantly, we will have $i_L(0^+) = i_L(0^-)$.

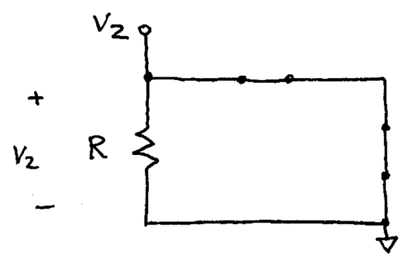
It follows that $v_2(0^+) = i_L(0^+)R = v_2(0^-)$.

$$\therefore \; v_2(0^+) = -12V \frac{R}{R + 10k\Omega}$$

This is the first piece of information we need for the general form of sol'n for $v_2(t)$:

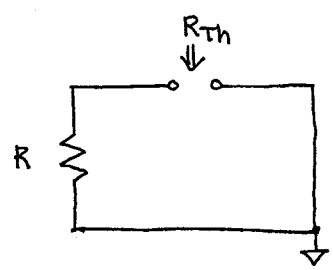
$$v_2(t) = v_2(t\to\infty) + \left[v_2(0^+) - v_2(t\to\infty)\right] e^{-t/\frac{L}{R_{Th}}}$$

For $t\to\infty$, the switch is closed and the current in L will decay toward zero. Also, the L will look like a wire.



We have $v_2(t\to\infty) = 0V$.

To find R_{Th} for the time constant, we remove L and look into the circuit from those terminals. (If there are independent sources, we turn them off.)



We see that $R_{Th} = R$. Now we can write an expression for $v_2(t)$:

$$v_2(t) = 0V + \left[v_2(0^+) - 0V\right] e^{-t/\frac{L}{R}}$$

or $$v_2(t) = \frac{-12VR}{R+10k\Omega} e^{-t/\frac{L}{R}}$$

We observe that $v_2(0^+) = v_2(0^-)$ means v_o will stay low initially, (i.e., $v_o = -12V$).

v_o goes high, (i.e., $v_o = +12V$), when $v_1 = v_2$ as v_2 approaches 0V from below.

From the graph of v_o, we have $v_1 = v_2$ at $t = 1\,\mu s$. Thus, we solve the following eq'n:

$$v_2(t=1\,\mu s) = -12V \frac{R}{R+10k\Omega} e^{-(t=1\mu s)/\frac{L}{R}} = v_1 = -9V$$

We have only one eq'n in two variables. Thus, the solution is not unique. Since R appears in several places in the eq'n, we simplify the problem by choosing R before solving the eq'n for L.

Earlier, we showed $R > 30\,k\Omega$. In practice, it is prudent to pick R large enough to a significant difference between $v_2(0^+)$ and v_1. This results in a smaller time constant, L/R, since v_2 has to climb further before $v_2 = v_1$. This, in turn, allows us to use a smaller L value. On the other hand, we want to avoid a very large R, (much larger than $1M\Omega$, for example), owing to possible problems with noise or with inaccuracies arising from using currents nearly as small as the minute current flowing into the op-amp.

We use $R = 110\,k\Omega$ for practicality and convenience.

Then $v_2(0^+) = -12V \cdot \frac{110k\Omega}{110k\Omega + 10k\Omega} = -11V$

Now we solve $v_2(t = 1\mu s) = -11V\, e^{-1\mu s / \frac{L}{110k\Omega}} = -9V.$

or $e^{-1\mu s / \frac{L}{110k\Omega}} = \frac{-9V}{-11V} = \frac{9}{11}$

or, taking ln of both sides, $\frac{-1\mu s}{L/110k\Omega} = \ln\left(\frac{9}{11}\right)$

or $L = \frac{-1\mu s \cdot 110k\Omega}{\ln\left(\frac{9}{11}\right)} = \frac{-110\ mH}{-200.7\ m}$

or $L \approx 0.55\ H$ $(L/R = 0.55H/110k\Omega = 5\mu s)$

This L value is rather large, meaning that we still need a large time constant.

Some thought shows that there are limits on how small L could be.

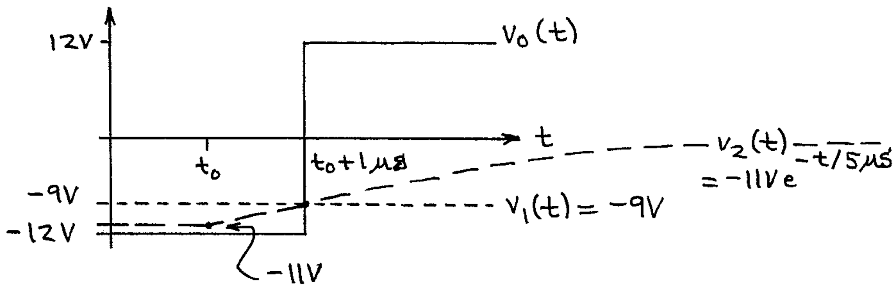
If we consider the limiting cases, we have $R = 30k\Omega$ min or $R \to \infty\ \Omega$ max.

For $R = 30k\Omega$ we get $L = \frac{-1\mu s \cdot 30k\Omega}{\ln\left(\frac{9}{9}\right)} = \infty\ H$ since $\ln(1) = 0$.

For $R \to \infty\Omega$ we get $L = \frac{-1\mu s \cdot \infty\Omega}{\ln\left(\frac{9}{12}\right)} = \infty\ H$

Thus, there is no escaping a large L.

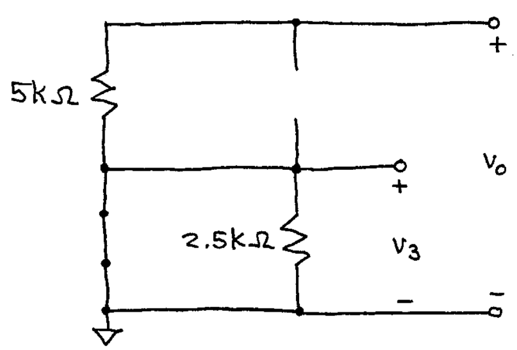
b) $v_1(t) = -9V$ for all time



c) $v_2(t) = -11V e^{-t/5\mu s}$ (shown on above plot)

d) To find v_3, we model diodes as wires when current flows in the forward direction, which is the direction of the "arrow" in the diode symbol. Otherwise, the diode is an open.

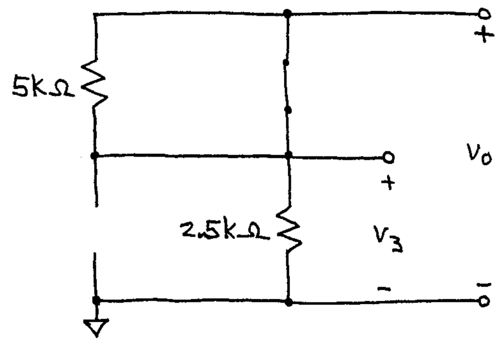
For $v_o = -12V$, our circuit model is



$v_3 = 0V$ since the diode on the lower left shorts v_3 to reference.

Thus, $v_3 = 0V$ whenever $v_o = -12V$.
In other words, $v_3 = 0V$ for $t < t_o + 1\mu s$.

For $v_o = +12V$, our circuit model is



$v_3 = +12V$ since the diode on the upper right shorts v_3 to v_o.

Thus, $v_3 = +12V$ for $t > t_o + 1\,\mu s$.