



a) Find the rms value of the periodic voltage shown.

sol'n: rms  $\equiv$  root mean square

- 1) First we have the (square) root:  $\sqrt{\quad}$
- 2) Inside that root, we have the mean (or average) which we compute as the area under the curve for one cycle divided by the length of one cycle (or period) of the waveform:

$$\sqrt{\frac{1}{T} \int_0^T dt}$$

$\uparrow$   
 area under curve  
 length of one period

Note: The function we integrate will be the square of waveform shown above and will be  $\geq 0$  everywhere. The average of the waveform shown above is zero.

Note: Think of the function we are taking the mean of as a hill of dirt. If we spread the dirt out so it is perfectly flat, the height of the dirt is the mean value of the function. The  $\int_0^T dt$  calculates how much dirt we have and  $\frac{1}{T}$  accounts for the length of the lot.

3) Inside that mean, we have the square of the waveform:

$$\sqrt{\frac{1}{T} \int_0^T V_g^2(t) dt}$$

So rms  $\Rightarrow$  root then mean then square.

$$V_g \text{ rms} = \sqrt{\frac{1}{120\mu\text{s}} \int_{0\mu\text{s}}^{120\mu\text{s}} V_g^2(t) dt}$$

We have to break the  $\int$  into sections. Within each section,  $V_g^2(t)$  is constant.

$$\begin{aligned} \int_0^{120\mu\text{s}} V_g^2(t) dt &= \int_0^{20\mu\text{s}} V_g^2(t) = 400 \text{ V}^2 dt + \int_{20\mu\text{s}}^{40\mu\text{s}} V_g^2(t) = 10 \text{ kV}^2 dt \\ &+ \int_{40\mu\text{s}}^{60\mu\text{s}} V_g^2(t) = 400 \text{ V}^2 dt + \int_{60\mu\text{s}}^{80\mu\text{s}} V_g^2(t) = 400 \text{ V}^2 dt \\ &+ \int_{80\mu\text{s}}^{100\mu\text{s}} V_g^2(t) = 10 \text{ kV}^2 dt + \int_{100\mu\text{s}}^{120\mu\text{s}} V_g^2(t) = 400 \text{ V}^2 dt \end{aligned}$$

Since the function is constant in each of these  $\int$ 's, we may calculate their value as width  $\cdot$  height (= area).

$$\begin{aligned} \therefore \int_0^{120\mu\text{s}} V_g^2(t) dt &= 20\mu\text{s} \cdot 400 \text{ V}^2 + 20\mu\text{s} \cdot 10 \text{ kV}^2 \\ &+ 20\mu\text{s} \cdot 400 \text{ V}^2 + 20\mu\text{s} \cdot 400 \text{ V}^2 \\ &+ 20\mu\text{s} \cdot 10 \text{ kV}^2 + 20\mu\text{s} \cdot 400 \text{ V}^2 \\ &= 20\mu\text{s} (4 \cdot 400 + 2 \cdot 10 \text{ k}) \text{ V}^2 \\ &= 20\mu\text{s} \cdot 21.6 \text{ k} \text{ V}^2 \\ &= 432 \text{ mV}^2 \text{ s} \end{aligned}$$

$$\text{mean value} = \frac{1}{120\mu\text{s}} 432 \text{ mV}^2 \text{ s} = 3.6 \text{ kV}^2$$

$$\text{rms value} = 60 \text{ V (rms)}$$

b) If waveform is applied to  $12\Omega$  resistor, find ave power dissipated.

sol'n: 
$$P = \frac{(V_{rms})^2}{R} = \frac{3600 V^2}{12 \Omega} = 300 W$$