CONCEPTUAL TOOLS

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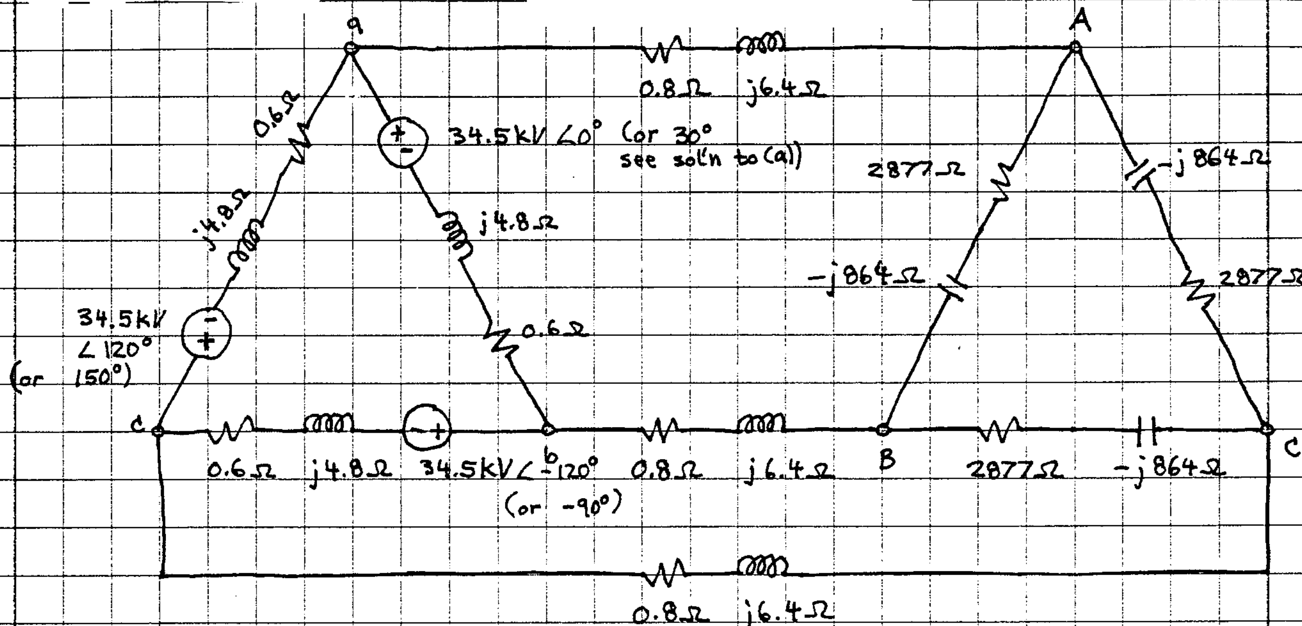
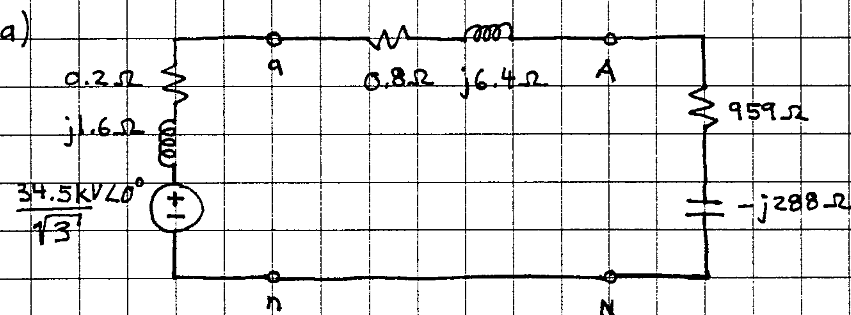
POWER ELECTRONICS
3-PHASE SYSTEMS
Example 2

Ref: Nilsson & Riedel, Rev 6, Prob 11.17



a) Construct single-phase equiv circuit
b) Calculate magnitude of line current
c) Calculate " " " voltage at terminals of load
d) " " " " " " " " source
e) " " " phase current at load
f) " " " " " in source

ans: a)



b) $|I_{aA}| = 19.9\text{ A}$
c) $|V_{AB}| = 34.6\text{ kV}$
d) $|V_{ab}| = 34.5\text{ kV}$
e) $|I_{AB}| = 11.5\text{ A}$
f) $|I_{ba}| = 11.5\text{ A}$

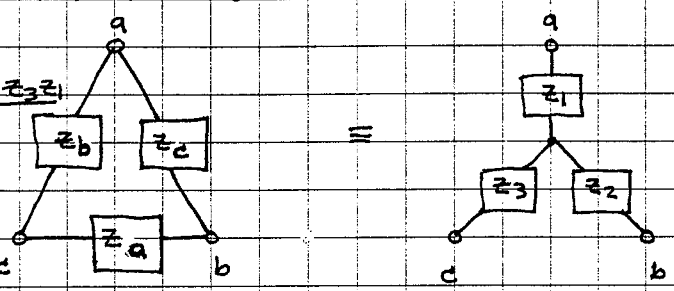
sol'n: a) We must transform the Δ source and Δ load into a Y source and Y load, respectively.

For the Δ load we use eqns 9.51 – 9.53 p 434 of Text:

$$Z_a = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}$$

$$Z_b = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}$$

$$Z_c = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}$$



$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$

If $Z_a = Z_b = Z_c$, then $Z_1 = Z_2 = Z_3 = \frac{Z_a^2}{3Z_a} = \frac{Z_a}{3}$.

∴ our Δ load transforms to Y load (per phase) of

$$Z_\phi = \frac{2877 - j864}{3} = 959 - j288 \ \Omega$$

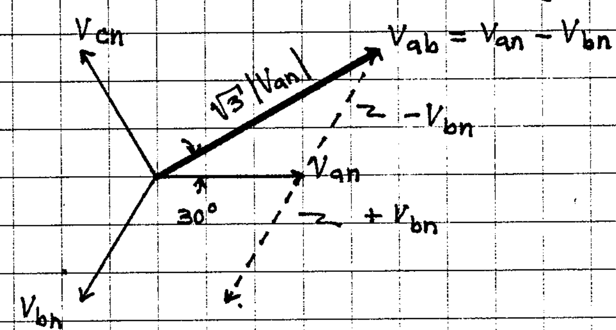
The source (or generator) internal impedances transform the same way.

∴ our Δ source internal Z's transform to Y source (per phase) Z's of $Z_g = \frac{0.6 + j4.8}{3} = 0.2 + j1.6 \ \Omega$.

One might suspect that we divide the source V's by 3 to get the Y source V's. This is <u>not</u> the case because each source is shifted by ±120° from the other two sources.

Instead, we observe that, with no load, the voltages at the Δ terminals are line voltages, (i.e. voltage diffences across a pair of lines), such as V_{ab}.

If we have a Y source, then we find $V_{ab} \equiv V_{an} - V_{bn}$ (for no load) from a phasor diagram:



Note: I'm assuming positive phase sequence since the problem statement in Text allows me to label a, b, c whichever way I desire.

From the diagram we see that $|V_{ab}| = \sqrt{3}\,|V_{an}|$.

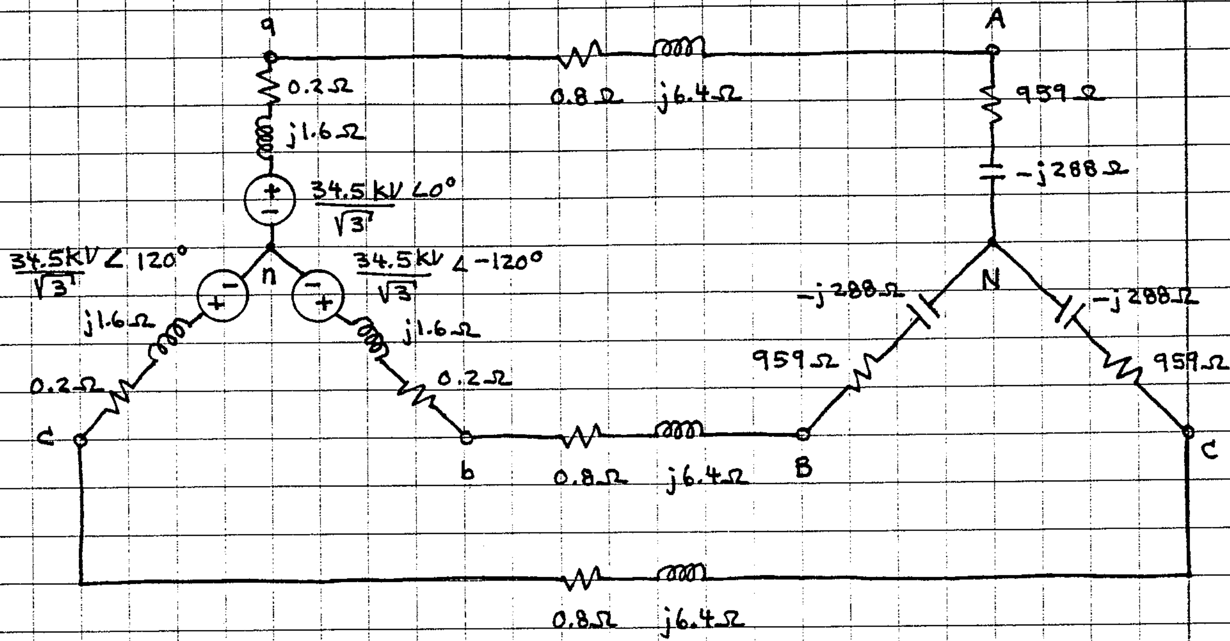
It follows that if we go the other direction, (from V_{ab} to V_{an}), we have:

Y source: $|V_{an}| = \dfrac{|V_{ab}|}{\sqrt{3}}$ i.e. $\dfrac{\Delta \text{ source } V}{\sqrt{3}}$

Note: We also have a $-30°$ phase shift when we transform from V_{ab} to V_{an} (i.e. from Δ source to Y source). We may, however, rotate our entire diagram any way we wish since the problem did not actually state which signal is at $\angle 0°$.

$\therefore$ We assume $\angle V_{an} = 0°$.

Since we don't transform them, our distribution line z's remain the same. Our complete circuit has become:

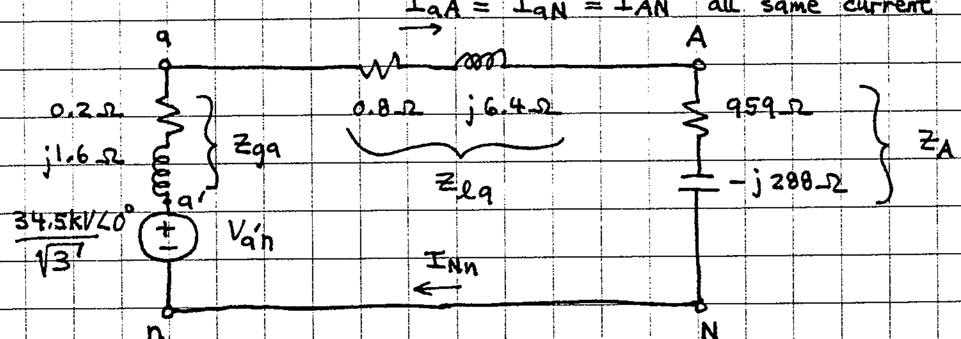


Note: Because the terminal characteristics of our source and load are exactly the same as before, we have exactly the same line voltages and currents as before.

Thus, we may use a single phase equivalent to the above circuit as a model for line voltages and currents in the original Δ-Δ configuration.

For the single-phase equivalent, we just use one leg of the above circuit:

$I_{aA} = I_{aN} = I_{AN}$ all same current



Note: This model gives correct I_{aA} line current, but not correct I_{Nn}. Use it for everything but I_{Nn}.

b) Magnitude of line current $\equiv |I_{aA}| = \left|\frac{V_{source}}{Z_{tot}}\right|$

$$= \left|\frac{34.5\text{ kV}\angle 0°/\sqrt{3}}{Z_g + Z_{la} + Z_A}\right| = \left|\frac{34.5\text{ kV}/\sqrt{3}}{0.2+j1.6+0.8+j6.4+959-j288\ \Omega}\right|$$

$$= \left|\frac{34.5\text{ kV}/\sqrt{3}}{960-j280\ \Omega}\right| = \left|\frac{34.5\text{ kV}/\sqrt{3}}{1\text{k}\Omega\angle -16.3°}\right| = \frac{34.5\text{kV}/\sqrt{3}}{1\text{k}\Omega}$$

$$= \frac{34.5\text{ A}}{\sqrt{3}} = 19.9\text{ A}$$

$$|I_{aA}| = 19.9\text{ A}$$

c) Magnitude of line voltage $\equiv |V_{AB}| = \sqrt{3}\,|V_{AN}|$
at terminals of load

(Same logic as $|V_{ab}| = \sqrt{3}\,|V_{an}|$ in part (a))

$$|V_{AN}| = |I_{aA}||Z_A| = 19.9\text{A}\cdot|959-j288| = 19.9\text{A}\cdot 1\text{k}\Omega = 19.9\text{kV}$$

$$\therefore\ |V_{AB}| = \sqrt{3}\,|V_{AN}| = \sqrt{3}\cdot 19.9\text{kV} = 34.5\text{ kV}$$

d) Magnitude of line voltage $\equiv |V_{ab}| = \sqrt{3}\,|V_{an}|$
at terminals of source

$$|V_{an}| = |I_{aA}||Z_A + Z_{la}| \quad \text{or} \quad |V_{an}| = |V_{a'n} - I_{aA} Z_{ga}|$$

Use 2nd form for answer guaranteed to be less than $V_{a'n}$.

$$|V_{an}| = \left|\frac{34.5\text{kV}}{\sqrt{3}} - \frac{34.5\text{ kV}}{\sqrt{3}}\frac{Z_{ga}}{Z_{ga}+Z_{la}+Z_A}\right|$$

$$= \left|\frac{34.5\text{kV}}{\sqrt{3}}\left(1 - \frac{Z_{ga}}{Z_{ga}+Z_{la}+Z_A}\right)\right| = \left|\frac{34.5\text{kV}}{\sqrt{3}}\frac{Z_{la}+Z_A}{Z_{ga}+Z_{la}+Z_A}\right|$$

$$= \frac{34.5\text{ kV}}{\sqrt{3}}\frac{0.8+j6.4+959-j288}{0.2+j1.6+0.8+j6.4+959-j288} = \frac{34.5\text{kV}}{\sqrt{3}}\left|\frac{959.8-j281.6}{960-j280}\right|$$

$$|V_{an}| = \frac{34.5\,\text{kV}}{\sqrt{3}} \left| \frac{1\text{k} \angle -16.35^\circ}{1\text{k} \angle -16.26^\circ} \right| = \frac{34.5\,\text{kV}}{\sqrt{3}} \frac{1\text{K}}{1\text{K}}$$

$$|V_{an}| = \frac{34.5\,\text{kV}}{\sqrt{3}}$$

$$\therefore\ |V_{ab}| = \sqrt{3}\,|V_{an}| = 34.5\,\text{kV}$$

Comment: $|z_{ga}|$ and $|z_{\ell a}| \ll |z_A|$ so only small part of total % drop is across z_{ga} and $z_{\ell a}$. In other words, the voltage across the load $\approx$ generator voltage.

e) The phase current refers to the original Δ load.

Phase current $\equiv$ current through one leg of Δ load
$\equiv I_{AB}$

For original Δ-Δ circuit we have $|I_{AB}| = \dfrac{|V_{AB}|}{|2877 - j864\,\Omega|}$

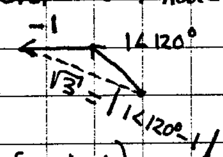
and we can use $|V_{AB}|$ from (b) (i.e. from single-phase model):

$$|I_{AB}| = \frac{|V_{AB}|}{|2877 - j864|} = \frac{|V_{AB}|}{|3 z_A|} = \frac{\sqrt{3}\,|V_{AN}|}{3\,|z_A|}$$

$$= \frac{\sqrt{3}}{3} \frac{|I_{aA}|\,|z_A|}{|z_A|} = \frac{|I_{aA}|}{\sqrt{3}} = \frac{19.9\,A}{\sqrt{3}} = 11.5\,A$$

$$|I_{AB}| = 11.5\,A$$

f) Phase current in source refers to original Δ circuit.
We want $|I_{ab}|$.
We have $I_{aA} = I_{ca} - I_{ab}$ (sum of currents out of node $= 0$)



$$I_{ca} = I_{ab} \cdot (1 \angle 120^\circ) \Rightarrow |I_{aA}| = |I_{ab}|\,|1 \angle 120^\circ - 1|$$

$$\therefore\ |I_{ab}| = |I_{aA}|/\sqrt{3} = 19.9\,A/\sqrt{3} = 11.5\,A \quad (= I_{AB} \text{ for } \Delta\text{-}\Delta)$$