EX: You are given the following information about events A, B, and C:

$$P(A) = 0.625 \qquad P(B) = 0.4 \qquad P(A \cap B) = 0.125$$

Which of the following statements *must* be true? Justify your answers.

a) $P(A \cap B') = 0.9$

b) $P(A \cup B) + P(A \cap B) = 1$

c) If $P(C) = 0.5$, then $P(A \cap C) > P(A \cap B)$

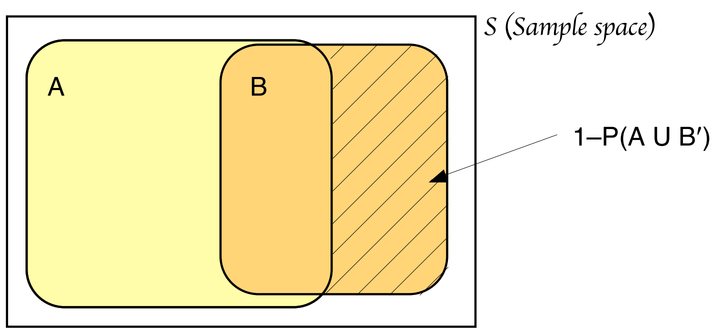
d) $P(A \cup B') > P(B \cup A')$

SOL'N: a) Need not be true. Actually, $P(A \cap B') = 0.9$ *cannot* be true. If this were true, we would have $P(A \cap B) + P(A \cap B') = 0.125 + 0.9 = P(A)$ [by Law of Total Probability] $= 1.025 > 1$. This is impossible.

b) Need not be true. Actually, $P(A \cup B) + P(A \cap B) = 1$ *cannot* be true. We can calculate $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.625 + 0.4 - 0.125 = 0.9$. Thus, $P(A \cup B) + P(A \cap B) = 0.9 + 0.125 = 1.025 \neq 1$. In general, $P(A \cup B) + P(A \cap B)$ may take on values in the range 0 to 2.

c) Need not be true. But for a missing = sign, this would be true. That is, if we had ≥ instead of >, then we could say the statement must be true. We observe that $P(A) = 0.625$ and $P(C) = 0.5$ implies that A and C must overlap by at least 0.125 in order for $P(A \cup C)$ to be ≤ 1. Thus, $P(A \cap C) \geq 0.125 = P(A \cap B)$. But for the missing =, we could say the statement must be true.

d) Must be true. We can calculate both quantities.
$P(A \cup B') = 1 - [1 - P(A \cup B')] = 1 - [P(B) - P(A \cap B)]$
$= 1 - [0.4 - 0.125] = 1 - 0.275 = 0.725$



$P(B \cup A') = 1 - [1 - P(B \cup A')] = 1 - [P(A) - P(A \cap B)]$
$= 1 - [0.625 - 0.125] = 1 - 0.5 = 0.5$

Thus, $P(A \cup B') = 0.725 > 0.5 = P(B \cup A')$.