Ex: A company has found that a certain fraction of the parts it orders are counterfeit. That fraction depends on which company the parts were ordered from. The parts are mixed together in a stockroom, so the selection of parts may be viewed as an experiment in probability. The following information is known.

$A \equiv$ event that parts selected are from company A

$B \equiv$ event that parts selected are from company B

$C \equiv$ event that parts selected are counterfeit

$P(A) = 0.35$

$P(B) = 0.25$

$P(C) = 0.10$

$P(A \cap C) = 0.08$

$P(B \cap C) = 0.01$

What is the probability of picking a part that is counterfeit and not from either company A or B? Note that we may write this probability as $P(C \cap (A \cap B)')$

Sol'n: Since we are calculating the probability of an intersection, we consider using the law of total probability. To do so, we need a partition of the sample space, S, of all possible outcomes. We may safely assume that A and B are mutually exclusive, since they are distinct companies. To complete the partition, we use the rest of S. That is, we use $(A \cap B)'$. Our partition is A, B, and $(A \cap B)'$.

Venn diagram:
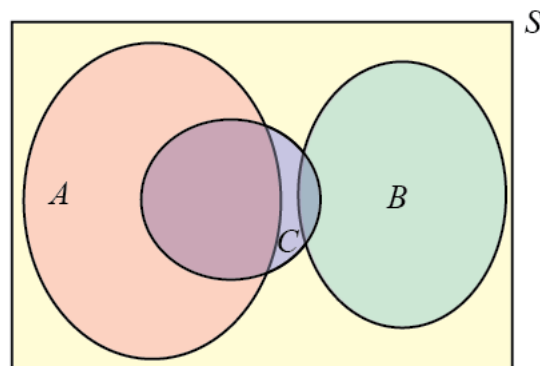
We want the area in C and between A and B in the Venn diagram. By the law of total probability, we use the following calculation of the probability of C:

$$P(C) = P(A \cap C) + P(B \cap C) + P(C \cap (A \cap B)')$$

We know the values of all the terms except the one we are looking for.

$$0.10 = 0.08 + 0.01 + P(C \cap (A \cap B)')$$

We solve for our unknown value to complete the solution.

$$P(C \cap (A \cap B)') = 0.10 - (0.08 + 0.01) = 0.01$$