Ex: Three companies, A, B, and C provide cell-phone coverage in a city. For a randomly chosen location in the city, the probability of coverage for the first two companies is as follows:

$$P(A) = 0.8 \qquad P(B) = 0.75$$

The following probabilities of coverage by company A or B or by B and C are also known:

$$P(A \cup B) = 0.9 \qquad P(B \cap C) = 0.45$$

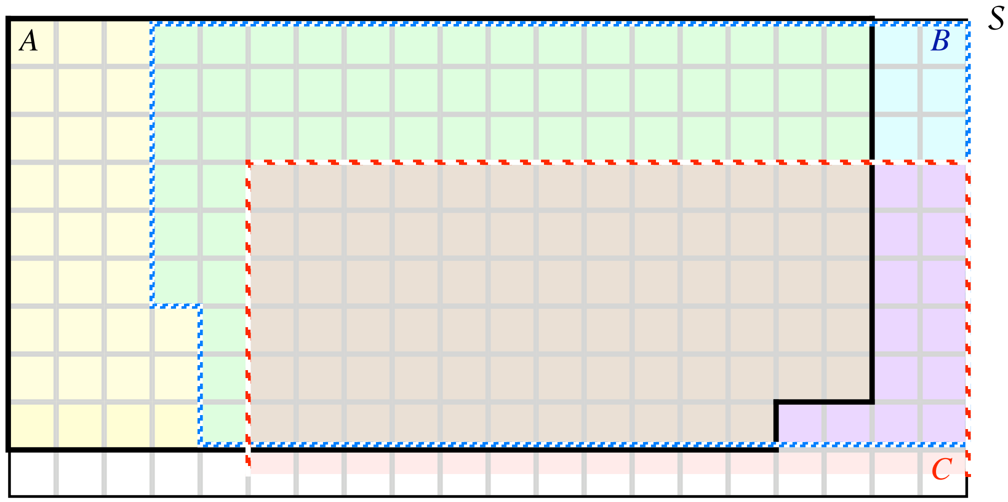
a) Find the probability, $P(A')$, of not having coverage from company A.

b) Find the probability, $P(A \cap B)$, of having coverage from both company A and company B.

c) Company A claims their probability of coverage, $P(A)$, is higher than company C's probability of coverage, $P(C)$. Determine whether this statement is true or false, and justify your answer.

d) Find the smallest possible value the probability, $P(B \cup C)$, of having coverage from company B or company C or both can be. (In case you have two cell phones...)

SOL'N: The Venn diagram below shows the information that is known about events A, B, and C. Areas in the diagram correspond to probabilities.



Because $P(C)$ is unknown, the area for C is shown open-ended.

We must be careful to draw only conclusions that follow from the information given in the problem. The exact value of $P(A \cap B \cap C)$, for example, is unknown, although the diagram shows it as 0.38.

a) For any event A, the probability of the complement of A, A', is one minus the probability of A:

$$P(A') = 1 - P(A)$$

Using numerical values yields the answer:

$$P(A') = 1 - P(A) = 1 - 0.8$$

NOTE: A and A' form a (total) partition. Together they account for all possibilities, (i.e., the entire sample space), and they are mutually exclusive.

b) To find $P(A \cap B)$, we use the equation for $P(A \cup B)$:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Rearranging the equation yields the value of $P(A \cap B)$:

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

Substituting numerical values given in the problem, we have the following result:

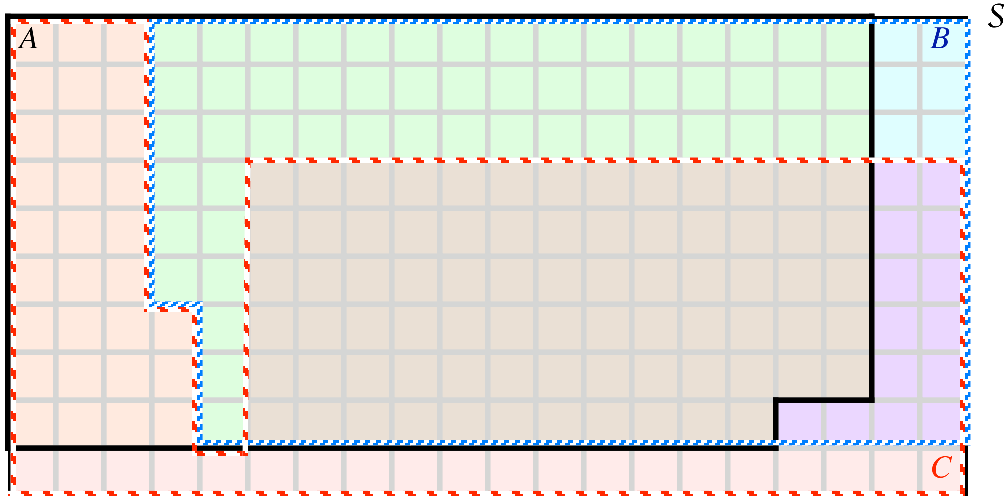
$$P(A \cap B) = 0.8 + 0.75 - 0.9 = 0.65$$

We could also use the Venn diagram and determine the area of the intersection of A and B. Counting the green and brown squares in the intersection gives a value of 130 squares/200 squares in the intersection of A and B. In other words, $P(A \cap B) = 0.65$.

c) The largest value that $P(C)$ may possibly have, given the information in the problem, occurs if C includes *all* the area outside of B. In other words, the largest possible value for $P(C)$ is $P(C) = P(B \cap C) + P(B')$:

$$\max\ P(C) = P(B \cap C) + 1 - P(B) = 0.45 + 1 - 0.75 = 0.70$$

Using the Venn Diagram, we would have the following picture:



Everything inside the dashed red line represents that maximum possible size of C. We count 140 squares lying in C out of the total of 200. In other words, the maximum possible value of C is 140/200 = 0.7.

d) Without further knowledge, the smallest possible value of $P(B \cup C)$ is the larger of $P(B)$ and $P(C)$. This follows because the union of two events includes both events. Thus, $P(B \cup C) \geq P(B)$ and $P(B \cup C) \geq P(C)$. Since $P(C)$ is unspecified, it is possible that C lies entirely in B. This yields the smallest possible value for $P(C)$ and, therefore, for $P(B \cup C)$. In that case, $P(B \cup C) = P(B)$, since having C inside B implies $P(C) > P(B)$:

$$\min P(B \cup C) = P(B) = 0.75$$