EX: Consider the problem of modeling the power usage of a light-rail system. We use the following notation:

$\boldsymbol{A}$ = train is *A*ccelerating or decelerating

$\boldsymbol{C}$ = train is *C*oasting

$\boldsymbol{T}$ = train is stopped

$\boldsymbol{W}$ = train is consuming more than 1 MW of power

We are given that $\{\boldsymbol{A}, \boldsymbol{C}, \boldsymbol{T}\}$ is a total partition of the sample space. The following information is also available:

$P(\boldsymbol{A}) = 4/12$

$P(\boldsymbol{T}) = 1/12$

$P(\boldsymbol{W} \mid \boldsymbol{A}) = 5/8$

$P(\boldsymbol{W} \mid \boldsymbol{T}) = 1/8$

$P(\boldsymbol{W}) = 7/24$

a) Calculate $P(\boldsymbol{C})$.

b) Calculate $P(\boldsymbol{W} \mid \boldsymbol{C})$.

SOL'N: a) Since $\{\boldsymbol{A}, \boldsymbol{C}, \boldsymbol{T}\}$ is a total partition of the sample space, we have

$$P(\boldsymbol{A}) + P(\boldsymbol{C}) + P(\boldsymbol{T}) = 1$$

or

$$P(\boldsymbol{C}) = 1 - (P(\boldsymbol{A}) + P(\boldsymbol{T})) = 1 - (4/12 + 1/12) = 7/12.$$

b) Using the law of total probability, we have

$$P(\boldsymbol{W}) = P(\boldsymbol{W} \cap \boldsymbol{A}) + P(\boldsymbol{W} \cap \boldsymbol{C}) + P(\boldsymbol{W} \cap \boldsymbol{T})$$

or

$$P(\boldsymbol{W}) = P(\boldsymbol{W}, \boldsymbol{A}) + P(\boldsymbol{W}, \boldsymbol{C}) + P(\boldsymbol{W}, \boldsymbol{T}).$$

We can expand the right side using the conditional probability formula:

$$P(\boldsymbol{A} , \boldsymbol{B}) = P(\boldsymbol{A} \mid \boldsymbol{B})\, P(\boldsymbol{B})$$

We use this to expand our previous equation:

$$P(\boldsymbol{W}) = P(\boldsymbol{W} \mid \boldsymbol{A})\, P(\boldsymbol{A}) + P(\boldsymbol{W} \mid \boldsymbol{C})\, P(\boldsymbol{C}) + P(\boldsymbol{W} \mid \boldsymbol{T})\, P(\boldsymbol{T}).$$

The only remaining unknown is $P(\boldsymbol{W} \mid \boldsymbol{C})$, which we can solve for:

$$P(\boldsymbol{W} \mid \boldsymbol{C}) = \frac{P(\boldsymbol{W}) - P(\boldsymbol{W} \mid \boldsymbol{A})P(\boldsymbol{A}) + P(\boldsymbol{W} \mid \boldsymbol{T})P(\boldsymbol{T})}{P(\boldsymbol{C})}$$

Using values given in the problem, we have

$$P(\boldsymbol{W} \mid \boldsymbol{C}) = \frac{\frac{7}{24} - \left(\frac{5}{8}\frac{4}{12} + \frac{1}{8}\frac{1}{12}\right)}{\frac{7}{12}} = \frac{1}{8}.$$