DEF: Bernoulli trials ≡ repeated identical experiments with independent outcomes that are 1 (success) or 0 (failure)

DEF: $p \equiv P(1) \equiv$ probability of success

DEF: $q \equiv P(0) \equiv$ probability of failure $= 1 - p$

EX: Flipping a fair coin constitutes a Bernoulli trial. We may define Heads as success, $P(\text{Heads}) = P(1) = p = 0.5$, and Tails as failure, $P(\text{Tails}) = P(0) = q = 1 - p = 0.5$.

DEF: Binomial distribution ≡ $P(m \text{ successes in } n \text{ trials}) = {}_nC_m \cdot p^m q^{n-m} = \dfrac{n!}{(n-m)!m!} p^m q^{n-m}$

NOTE: The binomial distribution is an example of combinatoric probabilities where the probability of a single outcome is $p^m q^{n-m}$.

EX: Suppose $p = P(1) = 0.4$ for a stream of bits in a communication system. Find the probability of 4 out of 6 bits being 1's.

SOL'N: There are ${}_6C_4$ patterns of 6 bits with four bits = 1. The patterns are 001111, 010111, 011011, ... , 111100.

The probability of a particular one of these patterns occurring as the outcome is p^4q^2. All of the patterns have the same probability, however, so our answer is given by the binomial distribution:

$$P(4 \text{ 1's in 6 bits}) = \frac{6!}{(6-4)!4!} p^4 q^2 = 15(0.4)^4(1-0.4)^2$$