EX: In redundant computing applications, (for example, space shuttle computers), computers must synchronize their clocks. One scheme for doing so involves each computer trying to identify itself as the master computer by issuing a binary signal at a random time interval after a start signal is received. This scheme works unless two computers try to issue their binary signals at nearly the same time. If the chances are 1 in 60 that two computers will issue their binary signals too close together, what are the odds that 3 out of 5 computers will issue their signals too close together? (To simplify the calculation, assume one computer actually issues its signals slightly ahead of the others, and find the probability that 2 out the remaining 4 computers issue their signals too close to the first one.)

SOL'N: We have Bernoulli trials with probability of success (ironically) $p = 1/60$. It follows that $q = 1 - p = 59/60$.

We use the binomial distribution to calculate the probability that 2 out 4 of the remaining computers will issue signals too close to the first computer's. This translates to 2 successes out of 4:

$$P(3 \text{ of } 5 \text{ computers signal too close together}) \approx$$

$$P(2 \text{ of } 4 \text{ computers signal too close to first computer}) =$$

$$_4C_2 \cdot p^2q^2 = \frac{4\cdot 3}{2\cdot 1}\cdot\left(\frac{1}{60}\right)^2\cdot\left(\frac{59}{60}\right)^2 = \frac{6\cdot 59^2}{60^4} \approx 0.00161$$

Note: An actual launch was once delayed when two computers failed to synchronize properly.