

EX: A computer on campus running the UNIX operating system has only 16 blocks of file space left. A student wishes to store a file that is 4 blocks long.

Give a count for each of the following:

- The number of ways to choose 4 blocks for the file if the order in which the blocks are used matters. (That is, we care which part of the file goes into which block.)
- The number of ways to choose 4 blocks for the file if the order in which the blocks are used does *not* matter. (The block allocation routine only needs to know which blocks are used and which are free.)
- The number of ways to choose 3 remaining blocks to store a second file *after* the 4-block file has been saved in 4 particular blocks. Here, the order in which the 3 blocks are chosen matters. Solve the problem using one particular set of 4 blocks that you are free to choose for the first file.
- The number of ways to choose two blocks that are separated by zero or one block if order does *not* matter and the initial 16 blocks are contiguous.

SOL'N: a) Use permutations since order matters: ${}_{16}P_4 = 16!/(16-4)! = 16 \cdot 15 \cdot 14 \cdot 13 = 43,680$.

b) Use combinations since order doesn't matter: ${}_{16}C_4 = {}_{16}P_4/4! = 16 \cdot 15 \cdot 14 \cdot 13 / (4 \cdot 3 \cdot 2 \cdot 1) = 1820$.

c) This is the same as part (a) but we start with 12 blocks left and use 3: ${}_{12}P_3 = 12!/(12-3)! = 12 \cdot 11 \cdot 10 = 1320$.

d) Here we may simply visualize all the possibilities. This is possible when we have a small number of possible outcomes. Even when we have many possible outcomes, it may be helpful to start listing possible outcomes as a guide to writing counting formulas.

We are using 16 blocks in a row from which we will pick two blocks in a row or two blocks separated by one block. The blocks next to each other are 1,2 or 2,3 or ... or 15,16. There are 15 of these. The blocks separated by one block are 1,3 or 2,4 or 3,5 or ... or 14,16. There are 14 of these. We also have not counted any sets twice since we are always moving our location in the set of 16 blocks. Thus, we have $15 + 14 = \boxed{29}$ ways to choose the two blocks.