EX: 7-segment LED displays are sometimes used to display a character instead one digit. How many unique patterns can two-digits of 7-segment LED's display? Count all the possible patterns of each segment being on or off, including all segments off and all segments on.

SOL'N: There are two ways to view this calculation: that patterns correspond to binary numbers with 0's representing segments that are off and 1's representing segments that are on, or that we can sum the counts of how many patterns have 0, 1, 2, ..., 14 segments on.

Viewing patterns as equivalent to binary numbers, we have 14 digit numbers yielding 2^{14} = 16,384 possible patterns.

If, instead, we count the number of patterns with 0, 1, 2, ..., 14 segments on, we use combinatoric coefficients (since we only care which segments are on—not which segment turns on first). We have 14 segments chosen m at a time:

$$\#\text{ patterns} = \sum_{m=0}^{14} {}_{14}C_m \quad \text{where} \quad {}_{14}C_m = \frac{14!}{(14-m)!m!}$$

or, since ${}_nC_m = {}_nC_{n-m}$

$$\#\text{ patterns} = 2\left(\frac{14!}{14!0!} + \frac{14!}{13!1!} + \frac{14!}{12!2!} + \frac{14!}{11!3!} + \frac{14!}{10!4!} + \frac{14!}{9!5!} + \frac{14!}{8!6!}\right) + \frac{14!}{7!7!}.$$

or

$$\#\text{ patterns} = 2(1 + 14 + 91 + 364 + 1001 + 2002 + 3003) + 3432$$

or

$$\#\text{ patterns} = 2() + 3432 = 2(6476) + 3432 = 16{,}384$$

NOTE: There is a connection between 2^n and combinatorial coefficients, namely the binomial expansion:

$$(1+x)^n = \sum_{i=0}^{n} {}_nC_i 1^{n-i} x^i$$

Plugging $x = 1$, $n = 14$ gives $2^{14} = {}_{14}C_0 + {}_{14}C_1 + \ldots + {}_{14}C_{14}$.