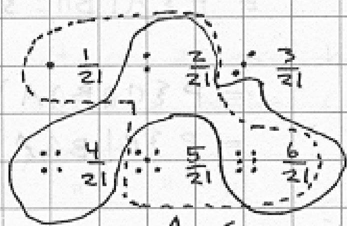
Conditional Probability

notn: $P\{A|B\} \equiv$ Probability of event A given event B occurred

ex: Loaded die



$B = \{2,4,6\}$

$A = \{1,2,5,6\}$

• *What is $P\{A\}$ given that we know B occurred*

• Sample space $\mathcal{S}$ becomes B

• Scale probs in B so they add up to one

⇒ divide original probs by $P\{B\} = \Sigma$ probs in B

here $P\{B\} = P\{2\} + P\{4\} + P\{6\} = \frac{2}{21} + \frac{4}{21} + \frac{6}{21} = \frac{4}{7}$

$$\therefore P\{2|B\} = \frac{P\{2\}}{4/7} = \frac{2}{21} \cdot \frac{7}{4} = \frac{1}{6}$$

$$P\{4|B\} = \frac{P\{4\}}{4/7} = \frac{4}{21} \cdot \frac{7}{4} = \frac{1}{3}$$

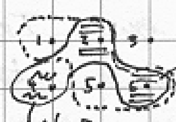
$$P\{6|B\} = \frac{P\{6\}}{4/7} = \frac{6}{21} \cdot \frac{7}{4} = \frac{1}{2}$$

• $P\{A|B\} = \frac{P\{\text{events in } A \text{ that are in } B = A \cap B\}}{P\{B\} = 4/7}$

$$= \frac{P\{2\} + P\{6\}}{4/7} = \frac{\frac{2}{21} + \frac{6}{21}}{4/7} = \frac{2}{3}$$

$$= P\{2|B\} + P\{6|B\}$$

tool: $P\{A|B\} = \frac{P\{A \cap B\}}{P\{B\}}$



ex: $P\{A|B\} + P\{A'|B\} = \frac{P\{A \cap B\}}{P\{B\}} + \frac{P\{A' \cap B\}}{P\{B\}} = \frac{P\{B\}}{P\{B\}} = 1$

ex: $P\{A|B'\} = \frac{P\{A \cap B'\}}{P\{B'\}} = \frac{P\{A\} - P\{A \cap B\}}{1 - P\{B'\}}$ = no simple relationship *tool:* Multiplication law

$$P\{A \cap B\} = P\{A \mid B\}\, P\{B\}$$ • *very useful*

ex: $P\{A \cap B \cap C\} = P\{A \mid B \cap C\}\, P\{B \cap C\}$

$= P\{A \mid B \cap C\}\, P\{B \mid C\}\, P\{C\}$

$= P\{C \cap B \cap A\}$

$= P\{C \mid B \cap A\}\, P\{B \mid A\}\, P\{A\}$ • *our choice*

• *recursive*

def: Independent Events A and B $\equiv$ $P\{A \mid B\} = P\{A\}$

note: If $P\{A \mid B\} = \dfrac{P\{A \cap B\}}{P\{B\}} = P\{A\}$

then $\dfrac{P\{A \cap B\}}{P\{A\}} = P\{B\}$, i.e. $\boxed{P\{B \mid A\} = P\{B\}}$,

and $\boxed{P\{A \cap B\} = P\{A\}\, P\{B\}}$

ex: independent events A, B, and C $\Rightarrow$ $P\{A \cap B \cap C\} = P\{A\} \cdot P\{B\} \cdot P\{C\}$

comments: Independent $\Rightarrow$ event gives no information about other event

Independent $\Rightarrow$ no physical interaction between events (sort of)

Independent $\Rightarrow$ can treat each event as being on separate axis

ex: Two dice red & green. Number on red die is independent of number on green die.