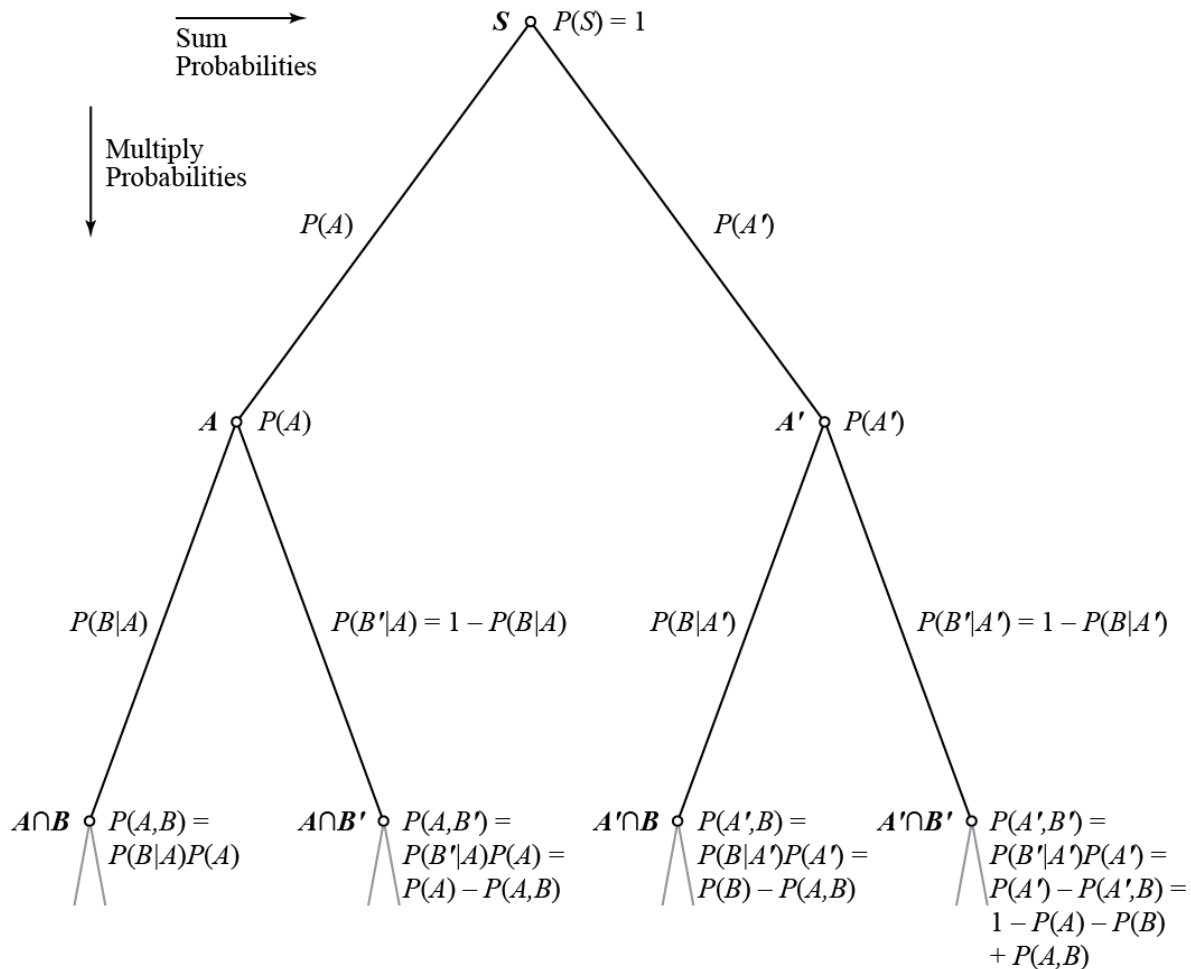


**TOOL:**

Multiplicative Probability Trees



Leaves form a partition: mutually exclusive, and union =  $S$  where  $S \equiv$  sample space

By Law of Total Probability:

$$P(A, B) + P(A, B') = P(A) \quad \text{and} \quad P(A, B) + P(A', B) = P(B)$$

$$P(A', B) + P(A', B') = P(A') \quad \text{and} \quad P(A, B') + P(A', B') = P(B')$$

By Law of Additive Probability:

$$P(A \cup B) = P(A) + P(B) - P(A, B)$$

$$P(A \cup B | C) = P(A|C) + P(B|C) - P(A, B|C)$$

By definition of Conditional Probability:

$$P(A|B) = P(A, B)/P(B) \quad \text{and} \quad P(B|A) = P(A, B)/P(A) \quad \text{and} \quad P(A|B) = P(B|A)P(A)/P(B)$$

$$A, B \text{ independent if and only if } P(A|B) = P(A) \Rightarrow P(A, B) = P(A)P(B)$$