

Ex: Consider flipping a fair coin until two Heads (H) or three Tails (T) are obtained. Find the set of possible outcomes and their probabilities.

SOL'N: We will have at most four coin flips before we have two Heads or three Tails. We list all possible sequences of four flips, which is the same as counting in binary (if we think of H as 0 and T as 1). Not all experiments would run to four flips; some would end sooner. Nevertheless, listing all four flips has the advantage of enabling us to calculate probabilities easily.

The number of possible sequences of four coin flips is $2^4 = 16$ since there are two possibilities at each step. All the sequences are equally likely if the coin is fair. So they all have probability $1/16$.

In the list, highlighting indicates where each sequence would stop because of two Heads or three Tails. Redundant entries in the list are in the same color. By counting the redundancies, we obtain the probabilities of each of the sequences.

HHHH 4x1/16	THHH 2x1/16
HHHT	THHT
HHTH	THTH 1/16
HHTT	THTT 1/16
HTHH 2x1/16	TTHH 1/16
HTHT	TTHT 1/16
HTTH 1/16	TTTH 2x1/16
HTTT 1/16	TTTT

So $P(HH) = 1/4$, $P(HTH) = 1/8$, $P(HTTH) = 1/16$, $P(HTTT) = 1/16$,

$P(THH) = 1/8$, $P(THTH) = 1/16$, $P(THTT) = 1/16$, $P(TTHH) = 1/16$,

$P(TTHT) = 1/16$, and $P(TTT) = 1/8$.