DEF: A and B are independent events $\equiv P(A|B) = P(A)$ (and $P(B|A) = P(B)$)
$\equiv$ knowing that event B has occurred doesn't change $P(A)$ and vice versa

TOOL: If A and B are independent events, then $P(A, B) = P(A)P(B)$.

EX: Consider rolling a pair of fair, six-sided dice. Let A be the event that the first die shows a 1, and let B be the event that the second die shows a 2. The number that shows on the first die has no influence on the number that shows on the second die. Thus, A and B are independent. Also, $P(A, B) = P(A)P(B) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$.

EX: Consider cards dealt from a deck of 52 playing cards. Is the probability of being dealt a king of hearts on the 3rd card, P(3rd card = K♥), independent of events relating to the first two cards dealt?

SOL'N: P(3rd card = K♥) is *dependent* on events relating to the first two cards dealt. P(3rd card = K♥ | 1st card = K♥) = 0, for example. (After the king of hearts is dealt, it's gone from the deck and cannot be dealt as the 3rd card).

We might be tempted to say that P(3rd card = K♥) is independent of the first two cards dealt when those cards are not the king of hearts. That would imply P(3rd card = K♥ | 1st 2 cards ≠ K♥) = P(3rd card = K♥), which is false. Calculation of the probabilities yields the following:

P(3rd card = K♥ | 1st 2 cards ≠ K♥) = 1/50

P(3rd card = K♥) = 1/52 (since nothing is known about 1st 2 cards)

The lesson to be learned is that sometimes either the concept of independence violates our intuitive notions or the mathematical expressions for independence fall short of capturing a probabilistic idea that we wish to express.

TOOL: If A and B are independent events, then the following events are also independent:

A and B' (where $B' \equiv$ complement of B, or not B)

A' and B

A' and B'