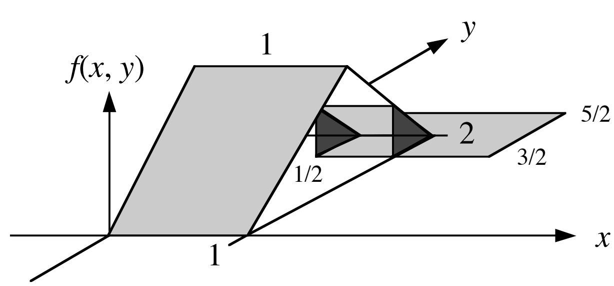
EX: Find $P(\frac{1}{2} < X < \frac{3}{2} \text{ and } Y > \frac{3}{2})$ for joint probability density function $f(x, y) = f_X(x) f_Y(y)$ where $f_X(x)$ is a uniform distribution on the interval [0, 1] and $f_Y(y)$ is a triangular distribution on the interval [0,2].

$$f_Y(y) = \begin{cases} 1 - |1 - y| & 0 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$$

SOL'N: The illustration, below, shows the 3-*D* shape of $f(x, y)$ and the volume of $f(x, y)$ that lies over the region $\frac{1}{2} < X < \frac{3}{2}$ and $Y > \frac{3}{2}$ and is equal to $P(\frac{1}{2} < X < \frac{3}{2} \text{ and } Y > \frac{3}{2})$.



The probability we are seeking is the volume of the small wedge. We may compute it by simple geometry as half the volume of a cube that measures 1/2 on each side:

$$P(\tfrac{1}{2} < X < \tfrac{3}{2} \text{ and } Y > \tfrac{3}{2}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$$

By restricting the limits of integration to the region of the *xy*-plane where is nonzero, we have an expression for the probability we seek:

$$P(\tfrac{1}{2} < X < \tfrac{3}{2} \text{ and } Y > \tfrac{3}{2}) = \int_{3/2}^{2} \int_{1/2}^{1} f(x,y)\,dx\,dy$$

or

$$P(\tfrac{1}{2} < X < \tfrac{3}{2} \text{ and } Y > \tfrac{3}{2}) = \int_{3/2}^{2} \int_{1/2}^{1} (2 - y)\,dy\,dx = \int_{3/2}^{2} (2 - y) x \Big|_{1/2}^{1} dy$$

or

$$P(\tfrac{1}{2} < X < \tfrac{3}{2} \text{ and } Y > \tfrac{3}{2}) = \int_{3/2}^{2} (2-y)\frac{1}{2}\,dy = y - \frac{y^2}{4}\Big|_{3/2}^{2}$$

or

$$P(\tfrac{1}{2} < X < \tfrac{3}{2} \text{ and } Y > \tfrac{3}{2}) = 2 - \frac{3}{2} - 1 + \frac{9}{16} = \frac{1}{16}$$