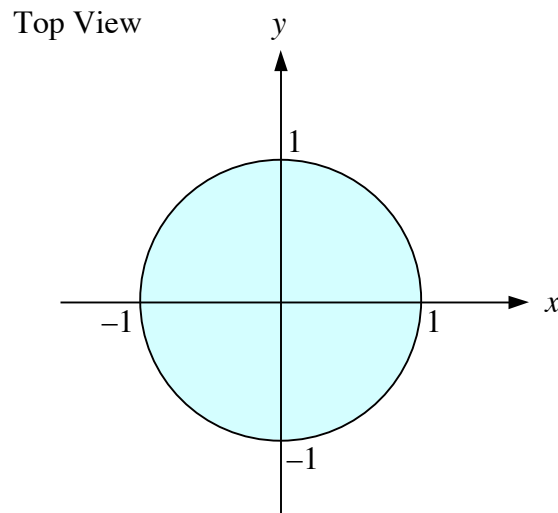


EX: A joint probability density function is defined as follows:

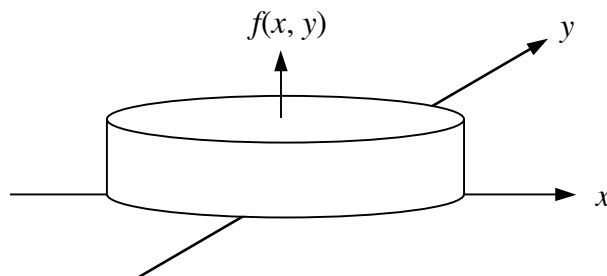
$$f(x, y) = \begin{cases} k & x^2 + y^2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- Sketch the shape of $f(x, y)$. (You may assume $k = 1$ for this sketch.)
- Calculate the value of k .

SOL'N: a) The region, $x^2 + y^2 \leq 1$, on which $f(x, y) \neq 0$ is called the support of $f(x, y)$. It is a circle of radius one, centered on the origin, as shown below.



Since $f(x, y) = k$ is constant on its support, $f(x, y)$ is a cylinder as shown below.



- b) The volume of $f(x, y)$ equals one. Since the volume is equal to the area of the support times height k , we have volume = $k \pi r^2$ where $r = 1$. It follows that $k = 1/\pi$. The illustration, below, shows the 3-dimensional shape of $f(x, y)$ with a height of $k = 1/\pi$.

