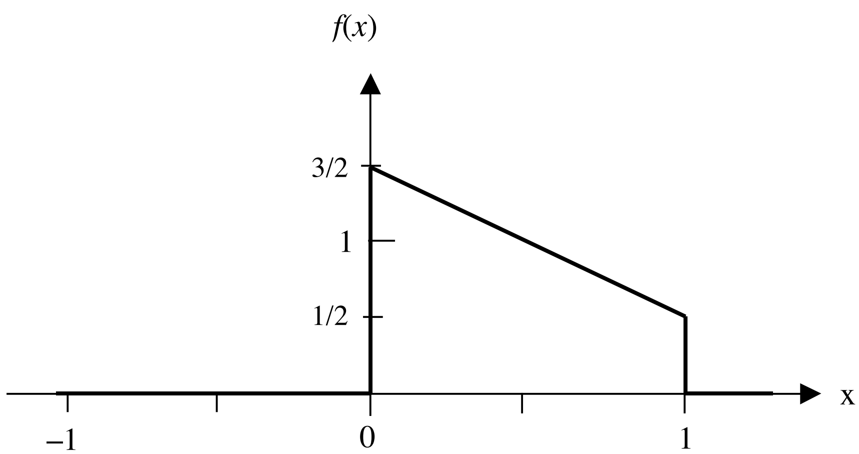
EX: The width of metal lines on an integrated circuit varies according to a distribution that falls off linearly above a certain minimum width, (which is $x = 0$). Find the expected value for excess line width given the following probability density function for excess line width:

$$f(x) = \begin{cases} \frac{1}{2} + (1 - x) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

SOL'N: By inspecting the plot of $f(x)$, below, we might estimate that the expected value or mean or μ for $f(x)$ is approximately 1/3.



To find the exact value of $E(x)$, (i.e., μ), we use the formula that defines it:

$$\mu \equiv E(x) = \int_{-\infty}^{\infty} x f(x) dx$$

Substituting for $f(x)$, and observing that $f(x)$ is nonzero only between 0 and 1, we have

$$\mu \equiv E(X) = \int_0^1 x \left[\frac{1}{2} + (1 - x) \right] dx = \int_0^1 \frac{3x}{2} dx - \int_0^1 x \cdot x dx .$$

After integrating, we have

$$\mu \equiv E(X) = \frac{3}{2} \frac{x^2}{2} \Bigg|_0^1 - \frac{x^3}{3} \Bigg|_0^1 = \frac{3}{2} \cdot \frac{1}{2} - \frac{1}{3} = \frac{5}{12} .$$