EX: Using Matlab® or pencil and paper, make an accurate plot of a standard gaussian distribution and answer the following questions:

a) What is the value of f(x) at x = 0?

b) At what value of x does $f(x) = 0.5$?

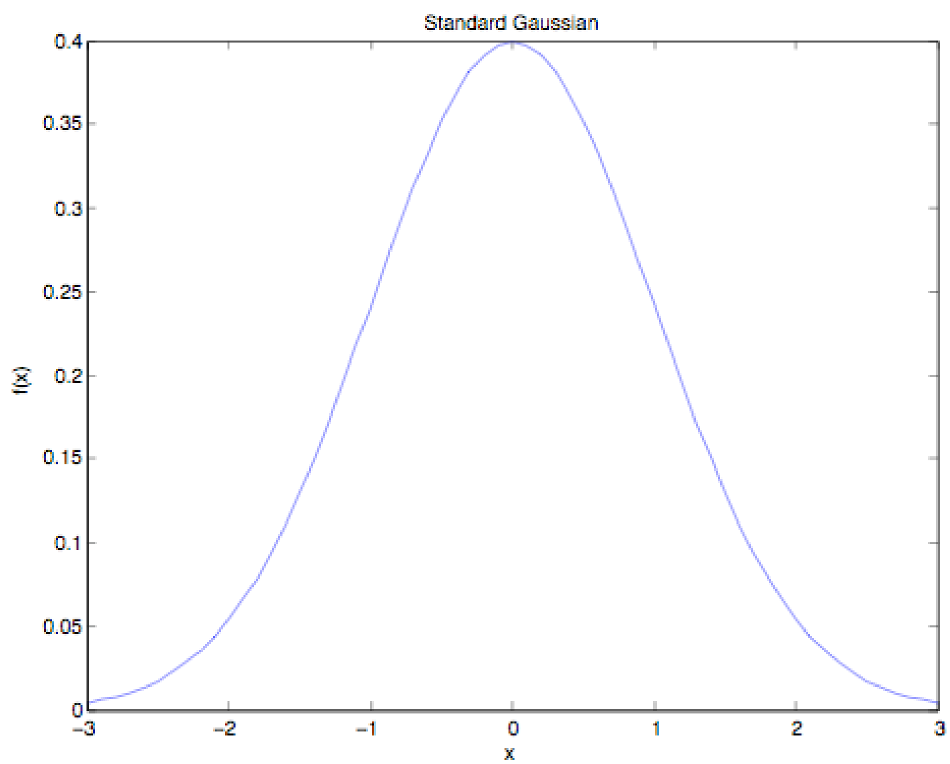
c) Estimate by eye the value of x for which $F(x) = 0.25$.

d) Use a table of area under the normal (i.e., gaussian) curve to find the value of x for which $F(x) = 0.25$.

e) Use a table of area under the normal (i.e., gaussian) curve to find $P(1 \le x \le 2)$.

SOL'N: a) The standard gaussian has $\mu = 0$ and $\sigma^2 = 1$:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2} = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$



The value of $f(x)$ at $x = 0$ is the constant term since $e^0 = 1$:

$$f(0) = \frac{1}{\sqrt{2\pi}} \approx 0.3989 = 0.4$$

b) From the plot, we observe that $f(x)$ never reaches a value of 0.5.

c) $F(x)$ is the cumulative distribution function, which is equal to the area under the probability function to the left of x. Thus, we are looking for the value of x where the area to the left of x is 1/4 of the total area of the probability density function, (since the total area under the probability density function is equal to one).

The author's estimate is $x \approx -3/4$.

d) Since we have a standard gaussian, we may use a table for the area under a standard gaussian directly. Note that such tables give values of $F(x)$. We use the table in reverse, however. We look for the value of $F(x) = 0.25$ in the table and then look at the value of x that corresponds to that $F(x)$. The value we obtain is –0.675 to three significant figures.

e) $P(1 \leq x \leq 2) = P(x \leq 2) - P(1 \leq x) = F(x = 2) - F(x = 1)$. Using a table for the area under a standard gaussian, we have

$F(x = 2) = 0.9772$ and $F(x = 2) = 0.8413$.

Thus,

$$P(1 \leq x \leq 2) = 0.9772 - 0.8413 = 0.1359.$$